

thm_2Ellist_2ENOT__LFINITE__DROP__LFINITE
(TMPAUB-
dKB5hVBa8xs1oRtyiGvJXUZhg7ngw)

October 26, 2020

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 4 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 5 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 6 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num ($

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (8)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (9)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2Ellist_rep A_27a \in \\ & (((ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum})^{ty_2Ellist_2Ellist A_27a}) \end{aligned} \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (11)$$

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (ap (c_2Emin_2E_3D_3D_3E (ap (ap (c_2Emin_2E_3D (2^{A_27a}$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Esum_2EABS_sum \\ & A_27a A_27b \in (((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (13)$$

Definition 13 We define c_Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_Esum_2EABS$
 Let $c_Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (14)$$

Definition 14 We define $c_Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_Eoption_2Eoption_ABS$

Definition 15 We define c_Ebool_2EF to be $(ap (c_Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 16 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let $c_Elist_2Elist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Elist_2Elist_abs A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Eenum_2Eenum}}) \quad (15)$$

Definition 17 We define $c_Elist_2ELCONS$ to be $\lambda A_27a : \iota.\lambda V0h \in A_27a.\lambda V1t \in (ty_2Elist_2Elist A_27a)$

Definition 18 We define c_Eone_2Eone to be $(ap (c_Emin_2E40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone))$

Definition 19 We define c_Ebool_2E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E3D_3D_3E V0t) c_Ebool_2E21 2))$

Definition 20 We define c_Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_Esum_2EABS$

Definition 21 We define $c_Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_Eoption_2Eoption_ABS A_27a) (ap$

Definition 22 We define c_Elist_2ELNIL to be $\lambda A_27a : \iota.(ap (c_Elist_2Elist_abs A_27a) (\lambda V0n \in ty_2Elist_2Elist A_27a))$

Definition 23 We define $c_Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E21 2) (\lambda V2t \in 2.V2t)))$

Definition 24 We define $c_Elist_2ELFINITE$ to be $\lambda A_27a : \iota.(\lambda V0a0 \in (ty_2Elist_2Elist A_27a).(ap (c_Elist_2Elist A_27a)$

Let $c_Elist_2ELDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Elist_2ELDROP A_27a \in (((ty_2Eoption_2Eoption (ty_2Elist_2Elist A_27a))^{(ty_2Elist_2Elist A_27a)})^{ty_2Eenum_2Eenum}) \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (22)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B)))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (30)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow 2.(((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (31)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0f \in (2^{A_{27a}}).(\forall V1v \in A_{27a}.((\forall V2x \in A_{27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (32)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0l \in (ty_2Ellist_2Ellist A_{27a}).((V0l = (c_2Ellist_2ELNIL A_{27a})) \vee (\exists V1h \in A_{27a}.(\exists V2t \in (ty_2Ellist_2Ellist A_{27a}).(V0l = (ap (ap (c_2Ellist_2ELCONS A_{27a}) V1h) V2t)))))) \quad (33)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow ((p (ap (c_2Ellist_2ELFINITE A_{27a}) (c_2Ellist_2ELNIL A_{27a}))) \Leftrightarrow True) \wedge (\forall V0h \in A_{27b}.(\forall V1t \in (ty_2Ellist_2Ellist A_{27b}).((p (ap (c_2Ellist_2ELFINITE A_{27b}) (ap (ap (c_2Ellist_2ELCONS A_{27b}) V0h) V1t))) \Leftrightarrow (p (ap (c_2Ellist_2ELFINITE A_{27b}) V1t)))))) \quad (34)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \forall A_{27c}.nonempty A_{27c} \Rightarrow ((\forall V0ll \in (ty_2Ellist_2Ellist A_{27a}).(ap (ap (c_2Ellist_2ELDROP A_{27a}) c_2Enum_2E0) V0ll) = (ap (c_2Eoption_2ESOME (ty_2Ellist_2Ellist A_{27a}) V0ll))) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap (ap (c_2Ellist_2ELDROP A_{27b}) (ap c_2Enum_2ESUC V1n)) (c_2Ellist_2ELNIL A_{27b})) = (c_2Eoption_2ENONE (ty_2Ellist_2Ellist A_{27b})))) \wedge (\forall V2n \in ty_2Enum_2Enum.(\forall V3h \in A_{27c}.(\forall V4t \in (ty_2Ellist_2Ellist A_{27c}).((ap (ap (c_2Ellist_2ELDROP A_{27c}) (ap c_2Enum_2ESUC V2n)) (ap (ap (c_2Ellist_2ELCONS A_{27c}) V3h) V4t)) = (ap (ap (c_2Ellist_2ELDROP A_{27c}) V2n) V4t)))))) \quad (35)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n)))))) \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE \\ & A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \end{aligned} \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (42)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\ & p\ V2r)) \vee (\neg(p\ V1q))) \wedge ((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (53)$$

Theorem 1

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0n \in \text{ty_2Enum_2Enum}. (\forall V1l \in (\text{ty_2Ellist_2Ellist } A_27a). (\forall V2t \in (\text{ty_2Ellist_2Ellist } A_27a). (((\neg(p (\text{c_2Ellist_2ELFINITE } A_27a) V1l))) \wedge (\text{ap } (\text{ap } (\text{c_2Ellist_2ELDROP } A_27a) V0n) V1l) = (\text{ap } (\text{c_2Eoption_2ESOME } (\text{ty_2Ellist_2Ellist } A_27a) V2t))) \Rightarrow \neg(p (\text{c_2Ellist_2ELFINITE } A_27a) V2t)))))))$$