

thm_2Ellist_2Eevery__thm
(TMZu4cWfhFkpVAUgSxLW4gdiQvZq8hi35rf)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 6 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (8)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (9)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (11)$$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (13)$$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone}) \quad (14)$$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_2ABS A_27a) (V0x))$

Definition 14 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E21_2) (\lambda V0t \in 2. V0t))$.

Definition 15 We define c_2Emin_2E40 to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap (c_2Ebool_2ECOND) (V1t1) (V2t2))))))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Eoption_2Eoption A_27a)^{ty_2Eenum_2Eenum}) \quad (15)$$

Definition 17 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist_abs A_27a) (V0h)$

Definition 18 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 19 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E7E))$

Definition 20 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS A_27a) (V0e))$

Definition 21 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_2ABS A_27a) (V0t))$

Definition 22 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist_abs A_27a. V0n))$

Definition 23 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27a}). (ap (c_2Ecombin_2Eo) (V0f) (V1g))$

Definition 24 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E40) (V0P))))$

Definition 25 We define $c_2Ellist_2Eexists$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\lambda V1a0 \in (ty_2Ellist_2Ellist_abs A_27a). (ap V1a0 (ap V0P))))$

Definition 26 We define $c_2Ellist_2Eevery$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). \lambda V1ll \in (ty_2Ellist_2Ellist_abs A_27a). (ap V1ll (ap V0P))$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (17)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(\\
& p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\
& nonempty A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\
& (\forall V2x \in A_27c.((ap (ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) \\
& V0f) V1g) V2x) = (ap V0f (ap V1g V2x))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1h \in \\
& A_27a.(\forall V2t \in (ty_2Ellist_2Ellist A_27a).(((p (ap (ap (\\
& c_2Ellist_2Eexists A_27a) V0P) (c_2Ellist_2ELNIL A_27a))) \Leftrightarrow False) \wedge \\
& ((p (ap (ap (c_2Ellist_2Eexists A_27a) V0P) (ap (ap (c_2Ellist_2ELCONS \\
& A_27a) V1h) V2t))) \Leftrightarrow ((p (ap V0P V1h)) \vee (p (ap (ap (c_2Ellist_2Eexists \\
& A_27a) V0P) V2t))))))
\end{aligned} \tag{23}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1h \in \\
& A_27a.(\forall V2t \in (ty_2Ellist_2Ellist A_27a).(((p (ap (ap (\\
& c_2Ellist_2Eevery A_27a) V0P) (c_2Ellist_2ELNIL A_27a))) \Leftrightarrow True) \wedge \\
& ((p (ap (ap (c_2Ellist_2Eevery A_27a) V0P) (ap (ap (c_2Ellist_2ELCONS \\
& A_27a) V1h) V2t))) \Leftrightarrow ((p (ap V0P V1h)) \wedge (p (ap (ap (c_2Ellist_2Eevery \\
& A_27a) V0P) V2t))))))
\end{aligned}$$