

thm_2Ellist_2Eexists__strongind (TMTkqhz8jFasNsG2KQHB14hrTFiF3z7oZj9)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_21` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))$

Definition 4 We define `c_2Ebool_2E_21` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow p \Rightarrow Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2E_7E}))$

Let `c_2Enum_2EZERO__REP` : ι be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \text{omega} \tag{1}$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{2}$$

Let `c_2Enum_2EABS__num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\text{omega}}) \tag{3}$$

Definition 7 We define `c_2Enum_2E0` to be $(\text{ap } c_2Enum_2EABS_num \text{ c_2Enum_2EZERO_REP})$.

Definition 8 We define `c_2Earithmetic_2EZERO` to be `c_2Enum_2E0`.

Let `c_2Enum_2EREP__num` : ι be given. Assume the following.

$$c_2Enum_2EREP_num \in (\text{omega}^{ty_2Enum_2Enum}) \tag{4}$$

Let `c_2Enum_2ESUC__REP` : ι be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\text{omega}^{\text{omega}}) \tag{5}$$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_2ESOME$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge p (ap P x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in \\ & ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \end{aligned} \quad (15)$$

Definition 17 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a.$

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 20 We define $c_2Ellist_2Eexists$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\lambda V1a0 \in (ty_2Ellist_2Ellist$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1y \in 2. (\forall V2z \in 2. (\forall V3w \in \\ & 2. (((((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \wedge (p V2z)) \Rightarrow \\ & ((p V1y) \wedge (p V3w)))))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1y \in 2. (\forall V2z \in 2. (\forall V3w \in \\ & 2. (((((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \vee (p V2z)) \Rightarrow \\ & ((p V1y) \vee (p V3w)))))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & (2^{A_27a}). ((\forall V2x \in A_27a. ((p (ap V0P V2x)) \Rightarrow (p (ap V1Q V2x)))) \Rightarrow \\ & ((\exists V3x \in A_27a. (p (ap V0P V3x))) \Rightarrow (\exists V4x \in A_27a. (p (\\ & ap V1Q V4x)))))) \end{aligned} \quad (21)$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1exists_27 \in \\
& \quad (2^{(ty_2Ellist_2Ellist\ A_27a)}).(((\forall V2h \in A_27a.(\forall V3t \in \\
& (ty_2Ellist_2Ellist\ A_27a).((p\ (ap\ V0P\ V2h)) \Rightarrow (p\ (ap\ V1exists_27 \\
& \quad (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a\ V2h)\ V3t)))))) \wedge (\forall V4h \in \\
& \quad A_27a.(\forall V5t \in (ty_2Ellist_2Ellist\ A_27a).(((p\ (ap\ (ap\ (\\
& \quad c_2Ellist_2Eexists\ A_27a\ V0P)\ V5t)) \wedge (p\ (ap\ V1exists_27\ V5t))) \Rightarrow \\
& (p\ (ap\ V1exists_27\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a\ V4h)\ V5t)))))) \Rightarrow \\
& \quad (\forall V6a0 \in (ty_2Ellist_2Ellist\ A_27a).((p\ (ap\ (ap\ (c_2Ellist_2Eexists \\
& \quad A_27a\ V0P)\ V6a0)) \Rightarrow (p\ (ap\ V1exists_27\ V6a0)))))))
\end{aligned}$$