thm_2Ellist_2Elinear__order__to__llist (TMHYB3sMyw6mqdEpwNAk2aJmxWiTfjyCGz9)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{5}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{6}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{7}$$

Definition 14 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{8}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{9}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{10}$$

Definition 15 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{11}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{12}$$

Definition 16 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (13)$$

Let $c\_2Ellist\_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELNTH\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (14)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (15)$$

Definition 17 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (16)$$

Definition 18 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 19 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 20 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 21 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 22 *We define* c_2Eset__relation_2Efinite__prefixes *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_$

Definition 23 *We define* c_2Eset__relation_2Eantisym *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27}$

Definition 24 *We define* c_2Eset__relation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_}$

Definition 25 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A}$

Definition 26 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 27 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod}$

Definition 28 *We define* c_2Eset__relation_2Elinear__order *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a}$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0lo \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1X \in (2^{A\_27a}).(((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V0lo)\ V1X)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Efinite\_\_prefixes}\ A\_27a\ A\_27a)\ V0lo)\ V1X))) \Rightarrow (\exists V2ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((V1X = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V3x)\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Enum\_2Enum)\ (\lambda V4i \in ty\_2Enum\_2Enum.(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V4i)\ V2ll))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3x)))))))) \wedge ((V0lo = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27a\ (ty\_2Epair\_2Eprod\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)\ 2))\ (\lambda V5x \in A\_27a.(\lambda V6y \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)\ 2)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V5x)\ V6y))\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Enum\_2Enum)\ (\lambda V7i \in ty\_2Enum\_2Enum.(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Enum\_2Enum)\ (\lambda V8j \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V7i)\ V8j))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V7i)\ V2ll))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V5x)))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V8j)\ V2ll))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V6y))))))))))))))) \wedge (\forall V9i \in ty\_2Enum\_2Enum.(\forall V10j \in ty\_2Enum\_2Enum.(\forall V11x \in A\_27a.((((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V9i)\ V2ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V11x)) \wedge ((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V10j)\ V2ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V11x))) \Rightarrow (V9i = V10j)))))))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V0s)\ V0s))) \tag{23}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0lo \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1X \in (2^{A\_27a}).(((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V0lo)\ V1X)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Efinite\_\_prefixes}\ A\_27a\ A\_27a)\ V0lo)\ V1X))) \Rightarrow (\exists V2ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((V1X = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V3x)\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Enum\_2Enum)\ (\lambda V4i \in ty\_2Enum\_2Enum.(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V4i)\ V2ll))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3x)))))))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ V0lo)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27a\ (ty\_2Epair\_2Eprod\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)\ 2))\ (\lambda V5x \in A\_27a.(\lambda V6y \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)\ 2)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V5x)\ V6y))\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Enum\_2Enum)\ (\lambda V7i \in ty\_2Enum\_2Enum.(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Enum\_2Enum)\ (\lambda V8j \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V7i)\ V8j))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V7i)\ V2ll))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V5x)))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V8j)\ V2ll))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V6y)))))))))))))))) \wedge (\forall V9i \in ty\_2Enum\_2Enum.(\forall V10j \in ty\_2Enum\_2Enum.(\forall V11x \in A\_27a.((((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V9i)\ V2ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V11x)) \wedge ((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V10j)\ V2ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V11x))) \Rightarrow (V9i = V10j)))))))))))$$