thm_2Ellist_2Elinear__order__to__llist__eq (TMMSHuKfn-QUCt4xLhkETjnvky1F8MGc6Wdx)

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Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (1)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.\mathsf{c\_2Ebool\_2ET})$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (2)$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (3)$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (4)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (5)$$

Let $c\_2Ellist\_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELNTH\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (6)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{7}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{8}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{9}$$

Definition 4 *We define* $\mathsf{c\_2Ecombin\_2EK}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 5 *We define* $\mathsf{c\_2Ebool\_2E\_21}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* $\mathsf{c\_2Ecombin\_2Eo}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{10}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{11}$$

Definition 7 *We define* $\mathsf{c\_2Epair\_2EUNCURRY}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b}$

Let $c\_2Eoption\_2EOPTION\_\_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})})^{(ty\_2Eoption\_2Eoption\ A\_27} \tag{12}$$

Let $c\_2Earithmetic\_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2EFUNPOW\ A\_27a \in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})}) \tag{13}$$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{14}$$

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{15}$$

Definition 8 *We define* c_2Ellist_2ELUNFOLD *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((ty\_2Eoption\_2Eoption$

Let $c\_2Ellist\_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELTAKE\ A\_27a \in (((ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a))^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{16}$$

Definition 9 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27a$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{17}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 13 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 14 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{18}$$

Definition 16 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A$

Definition 17 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod}$

Definition 18 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 19 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 20 *We define* c_2Eset__relation_2Eminimal__elements *to be* $\lambda A\_27a : \iota.\lambda V0xs \in (2^{A\_27a}).\lambda V1r \in$

Let $c\_2Epred\_\_set\_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epred\_\_set\_2ECHOICE\ A\_27a \in (A\_27a^{(2^{A\_27a})}) \tag{19}$$

Definition 21 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 22 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 23 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 24 *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{20}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{21}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{22}$$

Definition 25 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{23}$$

Definition 26 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 27 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 28 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 29 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 30 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 31 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 32 *We define* c_2Ellist_2Elinear__order__to__list__f *to be* $\lambda A\_27a : \iota.\lambda V0lo \in (2^{(ty\_2Epair\_2Eprod\ A}$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{24}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{25}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{26}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{27}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{28}$$

Definition 33 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 34 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 35 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 36 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{29}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{30}$$

Definition 37 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{31}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{32}$$

Definition 38 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 39 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Eoption\_2EOPTION\_\_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ (ty\_2Eoption\_2Eoption\ A\_27a))}) \tag{33}$$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{34}$$

Let $c\_2Eoption\_2EIS\_\_NONE : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eoption\_2EIS\_\_NONE\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{35}$$

Let $c\_2Eoption\_2EIS\_\_SOME : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{36}$$

Let $c\_2Eoption\_2ETHE : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{37}$$

Definition 40 *We define* c_2Epair_2Epair__CASE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair\_$

Definition 41 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 42 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 43 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 44 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{38}$$

Definition 45 *We define* c_2Epred__set_2EDELETE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (a$

Let $c\_2Epred\_\_set\_2ECARD : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Epred\_\_set\_2ECARD\ A\_27a \in (ty\_2Enum\_2Enum^{(2^{A\_27a})}) \tag{39}$$

Definition 46 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 47 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 48 *We define* c_2Epred__set_2EPSUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

Definition 49 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Definition 50 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 51 *We define* c_2Eset__relation_2Eantisym *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27}$

Definition 52 *We define* c_2Eset__relation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_}$

Definition 53 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 54 *We define* c_2Epred__set_2ESING *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_3F A_2

Definition 55 *We define* c_2Eset__relation_2Efinite__prefixes *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_}$

Definition 56 *We define* c_2Eset__relation_2Elinear__order *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a}$

Assume the following.

$$((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Enum\_2E0})) \tag{40}$$

Assume the following.

$$((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ \mathsf{c\_2Earithmetic\_2EZERO}) = \mathsf{c\_2Enum\_2E0}) \tag{41}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = \mathsf{c\_2Enum\_2E0}) \lor (\exists V1n \in ty\_2Enum\_2Enum.(V0m = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))))) \tag{42}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))))) \tag{43}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V0m))\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V1n)))))) \tag{44}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((V0m = V1n) \lor ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)) \lor (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1n)\ V0m)))))) \tag{45}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0m)) \Leftrightarrow ((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V0m)) = V0m))) \tag{46}$$

Assume the following.

$$True \tag{47}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1y \in A\_27a.((ap\ (\lambda V2x \in A\_27a.(ap\ V0f\ V2x))\ V1y) = (ap\ V0f\ V1y)))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{52}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \wedge (p\ V1t2)) \Leftrightarrow ((p\ V1t2) \wedge (p\ V0t1))))) \tag{53}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{54}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{55}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{56}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{57}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g)\Leftrightarrow(\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{61}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1)\wedge((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{64}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{65}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg (p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27)))))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{68}$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v)) \Leftrightarrow True)) \tag{69}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}).(\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{70}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27b).(((ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) \wedge ((ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V0h)\ V1t)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27b)\ V0h)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27b)\ V1t)))))) \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a))^{A\_27b}).(\forall V1x \in A\_27b.(\forall V2n \in ty\_2Enum\_2Enum. (((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V1x)) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)\ A\_27a)\ (c\_2Epair\_2ESND\ A\_27b\ A\_27a))\ (ap\ V0f\ V1x))) \wedge ((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V1x)) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ V0f\ V1x))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (\lambda V3v \in (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a).(ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ (ty\_2Eoption\_2Eoption\ A\_27a)\ A\_27b\ A\_27a)\ V3v)\ (\lambda V4tx \in A\_27b.(\lambda V5hx \in A\_27a.(ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V2n)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V4tx)))))))))))) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a))^{A\_27b}).(\forall V1x \in A\_27b.(\forall V2n \in ty\_2Enum\_2Enum. (((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V1x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge ((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V1x)) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a)))\ (ap\ V0f\ V1x))\ (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27a)))\ (\lambda V3v \in (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a).(ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a))\ A\_27b\ A\_27a)\ V3v)\ (\lambda V4tx \in A\_27b.(\lambda V5hx \in A\_27a.(ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5hx))\ (ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ V2n)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V4tx))))))))))))) \tag{73}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{74}$$

Assume the following.

$$
\begin{array}{c}
((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap \\
(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum. \\
((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge \\
((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum. \\
(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (\\
ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum. \\
((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP \\
\mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
(ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) = \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge \\
((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n)) \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c\_2Enum\_2ESUC} \\
\mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.(\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \\
\mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE} \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum. \\
(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge \\
((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum. \\
(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge \\
((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E} \\
\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}
\end{array}
$$

Assume the following.

$$(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Earithmetic\_2EZERO}) = \mathsf{c\_2Earithmetic\_2EZERO})\wedge (((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})) = \mathsf{c\_2Earithmetic\_2EZERO})\wedge((\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1n))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1n))))\wedge(\forall V2n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n)))))))) \tag{76}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{77}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ (ap\ V0f\ V1x)))))\wedge(\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V2f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))) \tag{78}$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\\
\forall V0e \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2e \in (\\
ty\_2Eoption\_2Eoption\ A\_27a).((\forall V3x \in A\_27a.(\forall V4y \in \\
A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ V4y)) \Leftrightarrow (V3x = V4y)))) \land ((\forall V5x \in A\_27a.((ap\ (c\_2Eoption\_2ETHE \\
A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V5x)) = V5x)) \land ((\forall V6x \in \\
A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ V6x)))) \land ((\forall V7x \in A\_27a.(\neg((ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ V7x) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \land ((\forall V8x \in A\_27a. \\
((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ V8x))) \Leftrightarrow True)) \land (((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a) \\
(\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \Leftrightarrow False) \land ((\forall V9x \in (ty\_2Eoption\_2Eoption \\
A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ V9x)) \Leftrightarrow (V9x = (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27a)))) \land ((\forall V10x \in (ty\_2Eoption\_2Eoption\ A\_27a).((\neg \\
(p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V10x))) \Leftrightarrow (V10x = (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27a)))) \land ((\forall V11x \in (ty\_2Eoption\_2Eoption\ A\_27a).((p \\
(ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V11x)) \Rightarrow ((ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ (ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ V11x)) = V11x))) \land ((\forall V12x \in \\
(ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE \\
A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V12x)\ (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27a))\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)) = V12x)) \land ((\forall V13x \in (\\
ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE \\
A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V13x)\ V13x)\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)) = V13x)) \land ((\forall V14x \in (ty\_2Eoption\_2Eoption\ A\_27a). \\
((p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ V14x)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE \\
A\_27a\ A\_27b)\ V14x)\ V0e)\ V1f) = V0e))) \land ((\forall V15x \in (ty\_2Eoption\_2Eoption \\
A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V15x)) \Rightarrow ((ap\ (ap \\
(ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ V15x)\ V0e)\ V1f) = (\\
ap\ V1f\ (ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ V15x))))) \land ((\forall V16x \in \\
(ty\_2Eoption\_2Eoption\ A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME \\
A\_27a)\ V16x)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ (\\
ty\_2Eoption\_2Eoption\ A\_27a))\ V16x)\ V2e)\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)) = \\
V16x))) \land ((\forall V17v \in A\_27b.(\forall V18f \in (A\_27b^{A\_27a}).(\\
(ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27a))\ V17v)\ V18f) = V17v))) \land ((\forall V19x \in A\_27a.(\forall V20v \in \\
A\_27b.(\forall V21f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE \\
A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V19x))\ V20v)\ V21f) = \\
(ap\ V21f\ V19x))))) \land ((\forall V22f \in (A\_27b^{A\_27a}).(\forall V23x \in \\
A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V22f)\ (\\
ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V23x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b) \\
(ap\ V22f\ V23x))))) \land ((\forall V24f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP \\
A\_27a\ A\_27b)\ V24f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27b))) \land (((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (\mathsf{c\_2Eoption\_2ENONE} \\
(ty\_2Eoption\_2Eoption\ A\_27a))) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \land \\
(\forall V25x \in (ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN \\
A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Eoption\_2Eoption\ A\_27a)) \\
V25x)) = V25x))))))))))))))))))))))))
\end{gathered}
\tag{79}
$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V2y \in A\_27b.(((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V2y)) \Leftrightarrow (\exists V3z \in A\_27a.((V1x = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3z)) \wedge (V2y = (ap\ V0f\ V3z)))))))) \tag{80}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27c}).(\forall V1g \in (A\_27c^{A\_27a}). (\forall V2x \in (ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27c\ A\_27b)\ V0f)\ (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27c)\ V1g)\ V2x)) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1g))\ V2x))))) \tag{81}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{82}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{83}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a. (\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r))))) \tag{84}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0x))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V0x)) = V0x)) \tag{85}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x))) \tag{86}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \tag{87}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y)) = (ap\ (ap\ V0f\ V1x)\ V2y))))) \tag{88}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0x \in A\_27b.(\forall V1y \in A\_27c.(\forall V2f \in ((A\_27a^{A\_27c})^{A\_27b}).((ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27c)\ V0x)\ V1y))\ V2f) = (ap\ (ap\ V2f\ V0x)\ V1y))))) \tag{89}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{90}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{91}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0y \in A\_27a.(\forall V1P \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0y)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V2x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V2x)\ (ap\ V1P\ V2x)))))) \Leftrightarrow (p\ (ap\ V1P\ V0y))))) \tag{92}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2P \in ((2^{A\_27b})^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ (ty\_2Epair\_2Eprod\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ 2))\ (\lambda V3x \in A\_27a.(\lambda V4y \in A\_27b.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ 2)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V3x)\ V4y))\ (ap\ (ap\ V2P\ V3x)\ V4y)))))))) \Leftrightarrow (p\ (ap\ (ap\ V2P\ V0x)\ V1y)))))) \tag{93}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))) \tag{94}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V0s)\ V0s))) \tag{95}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \lor (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{96}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{97}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V1t))\ V0s)))) \land (\forall V2s \in (2^{A\_27a}).(\forall V3t \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V3t)\ V2s))\ V2s))))) \tag{98}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t)))))))) \tag{99}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = V1y) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s))))))) \tag{100}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).(\neg((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ V1s) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))) \tag{101}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y))))))) \tag{102}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27a)\ V1s)\ V0x))\ V1s)))) \tag{103}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V1s)) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27a)\ V1s)\ V0x)) = V1s)))) \tag{104}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((\neg(V0s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ (ap\ (c\_2Epred\_\_set\_2ECHOICE\ A\_27a)\ V0s))\ V0s)))) \tag{105}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))) \Leftrightarrow (V0x = V1y)))) \tag{106}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \Leftrightarrow (V0x = V1y)))) \tag{107}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Epred\_\_set\_2ECHOICE\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) = V0x)) \tag{108}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \tag{109}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27a)\ V1s)\ V0x))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1s))))) \tag{110}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Epred\_\_set\_2ECARD\ A\_27a)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) = \mathsf{c\_2Enum\_2E0}) \tag{111}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1x \in A\_27a.((ap\ (c\_2Epred\_\_set\_2ECARD\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1x)\ V0s)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0s))\ (ap\ (c\_2Epred\_\_set\_2ECARD\ A\_27a)\ V0s))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (c\_2Epred\_\_set\_2ECARD\ A\_27a)\ V0s))))))) \tag{112}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1x \in A\_27a.((ap\ (c\_2Epred\_\_set\_2ECARD\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27a)\ V0s)\ V1x)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0s))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Epred\_\_set\_2ECARD\ A\_27a)\ V0s))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ (c\_2Epred\_\_set\_2ECARD\ A\_27a)\ V0s)))))) \tag{113}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1t \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V1t)\ V0s)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (c\_2Epred\_\_set\_2ECARD\ A\_27a)\ V1t))\ (ap\ (c\_2Epred\_\_set\_2ECARD\ A\_27a)\ V0s))))))) \tag{114}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(2^{A\_27a})}).((\forall V1x \in (2^{A\_27a}).((\forall V2y \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPSUBSET}\ A\_27a)\ V2y)\ V1x)) \Rightarrow (p\ (ap\ V0P\ V2y)))) \Rightarrow ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1x)) \Rightarrow (p\ (ap\ V0P\ V1x))))) \Rightarrow (\forall V3x \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V3x)) \Rightarrow (p\ (ap\ V0P\ V3x)))))) \tag{115}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) \Leftrightarrow (V0m = V1n)))) \tag{116}$$

Assume the following.

$$(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0m \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m)) = V0m))) \tag{117}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Enum\_2E0})))) \tag{118}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)))) \tag{119}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y))\ V2r)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Edomain}\ A\_27a\ A\_27b)\ V2r))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V1y)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erange}\ A\_27b\ A\_27a)\ V2r)))))))) \tag{120}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1s \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V0r)\ V1s))\ V0r)))) \tag{121}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).(\forall V1s \in (2^{A\_27b}).(\forall V2r\_27 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). (\forall V3s\_27 \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Efinite\_\_prefixes}\ A\_27a\ A\_27b)\ V0r)\ V1s)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V2r\_27)\ V0r)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27b)\ V3s\_27)\ V1s)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Efinite\_\_prefixes}\ A\_27a\ A\_27b)\ V2r\_27)\ V3s\_27))))))))) \tag{122}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1lo \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Eminimal\_\_elements}\ A\_27a)\ V0s)\ V1lo))\ V0s)))) \tag{123}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V2s\_27 \in (2^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V1r)\ V0s)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V1r)\ V2s\_27))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V2s\_27))))))) \tag{124}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0lo \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1X \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V0lo)\ V1X)) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Edomain}\ A\_27a\ A\_27a)\ V0lo))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erange}\ A\_27a\ A\_27a)\ V0lo)) = V1X)))) \tag{125}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0lo \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1X \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V0lo)\ V1X)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1X)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V2x)\ V2x))\ V0lo))))))) \tag{126}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0lo \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1X \in (2^{A\_27a}).(\forall V2x \in A\_27a.(\forall V3y \in A\_27a. ((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V0lo)\ V1X)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V2x)\ V3y))\ V0lo)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1X)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3y)\ V1X))))))))) \tag{127}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1X \in (2^{A\_27a}).(\forall V2lo \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V3y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V1X)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V2lo)\ V1X)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3y)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Eminimal\_\_elements}\ A\_27a)\ V1X)\ V2lo))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V3y)\ V0x))\ V2lo))))))))) \tag{128}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V2s \in (2^{A\_27a}). (\forall V3s\_27 \in (2^{A\_27a}).(((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V1r)\ V2s)) \land ((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Efinite\_\_prefixes}\ A\_27a\ A\_27a)\ V1r)\ V2s)) \land ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V3s\_27)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V3s\_27)\ V2s))))) \Rightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESING}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Eminimal\_\_elements}\ A\_27a)\ V3s\_27)\ V1r)))))))) \tag{129}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0lo \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}).\\
(\forall V1X \in (2^{A_27a}).(((p\ (ap\ (ap\ (\mathsf{c_2Eset__relation_2Elinear__order}\\
A_27a)\ V0lo)\ V1X)) \wedge (p\ (ap\ (ap\ (\mathsf{c_2Eset__relation_2Efinite__prefixes}\\
A_27a\ A_27a)\ V0lo)\ V1X))) \Rightarrow (\exists V2ll \in (ty_2Ellist_2Ellist\\
A_27a).((V1X = (ap\ (c_2Epred__set_2EGSPEC\ A_27a\ A_27a)\ (\lambda V3x \in\\
A_27a.(ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27a\ 2)\ V3x)\ (ap\ (\mathsf{c_2Ebool_2E_3F}\\
ty_2Enum_2Enum)\ (\lambda V4i \in ty_2Enum_2Enum.(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\\
(ty_2Eoption_2Eoption\ A_27a))\ (ap\ (ap\ (c_2Ellist_2ELNTH\ A_27a)\\
V4i)\ V2ll))\ (ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)\ V3x)))))))) \wedge ((V0lo =\\
(ap\ (c_2Epred__set_2EGSPEC\ (ty_2Epair_2Eprod\ A_27a\ A_27a)\ (ty_2Epair_2Eprod\\
A_27a\ A_27a))\ (ap\ (\mathsf{c_2Epair_2EUNCURRY}\ A_27a\ A_27a\ (ty_2Epair_2Eprod\\
(ty_2Epair_2Eprod\ A_27a\ A_27a)\ 2))\ (\lambda V5x \in A_27a.(\lambda V6y \in\\
A_27a.(ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ (ty_2Epair_2Eprod\ A_27a\ A_27a)\\
2)\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27a\ A_27a)\ V5x)\ V6y))\ (ap\ (\mathsf{c_2Ebool_2E_3F}\\
ty_2Enum_2Enum)\ (\lambda V7i \in ty_2Enum_2Enum.(ap\ (\mathsf{c_2Ebool_2E_3F}\\
ty_2Enum_2Enum)\ (\lambda V8j \in ty_2Enum_2Enum.(ap\ (ap\ \mathsf{c_2Ebool_2E_2F_5C}\\
(ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V7i)\ V8j))\ (ap\ (ap\ \mathsf{c_2Ebool_2E_2F_5C}\\
(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (ty_2Eoption_2Eoption\ A_27a))\ (ap\ (ap\ (\\
c_2Ellist_2ELNTH\ A_27a)\ V7i)\ V2ll))\ (ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)\\
V5x)))\ (ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (ty_2Eoption_2Eoption\ A_27a))\ (\\
ap\ (ap\ (c_2Ellist_2ELNTH\ A_27a)\ V8j)\ V2ll))\ (ap\ (\mathsf{c_2Eoption_2ESOME}\\
A_27a)\ V6y)))))))))))))) \wedge (\forall V9i \in ty_2Enum_2Enum.(\forall V10j \in\\
ty_2Enum_2Enum.(\forall V11x \in A_27a.((((ap\ (ap\ (c_2Ellist_2ELNTH\\
A_27a)\ V9i)\ V2ll) = (ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)\ V11x)) \wedge ((ap\ (\\
ap\ (c_2Ellist_2ELNTH\ A_27a)\ V10j)\ V2ll) = (ap\ (\mathsf{c_2Eoption_2ESOME}\\
A_27a)\ V11x))) \Rightarrow (V9i = V10j)))))))))))$$