thm_2Ellist_2Elnth_some_down_closed (TMWS-gBaZxo4MnMS1H3smV5bJWYRsB2pxWw4)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A$.**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A$

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (3)$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 9 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (5)$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 11 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 12 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $ty\_2Eoption\_2Eoption : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{8}$$

Let $ty\_2Ellist\_2Ellist : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{9}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in$$
$$(((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{10}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{11}$$

Definition 13 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum$$
$$A0\ A1) \tag{12}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum$$
$$A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{13}$$

Definition 14 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (\text{c\_2Esum\_2EABS}$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in$$
$$((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{14}$$

Definition 15 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_$

Definition 16 *We define* c_2Ebool_2EF *to be* $(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 17 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in \\ ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \quad (15)$$

Definition 18 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A.$

Definition 19 *We define* c_2Eone_2Eone *to be* $(ap\ (\textsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 20 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \textsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \textsf{c\_2Ebool\_2E}$

Definition 21 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 22 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 23 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty\_$

Let $c\_2Ellist\_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELNTH\ A\_27a \in (((ty\_2Eoption\_2Eoption \\ A\_27a)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum})$$

$$(16)$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (17)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (18)$$

Definition 24 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 25 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 26 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 27 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2E}$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(19)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(20)$$

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Definition 28 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 29 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 30 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = \text{c\_2Enum\_2E0})\lor(\exists V1n \in \\ ty\_2Enum\_2Enum.(V0m = (ap\ \text{c\_2Enum\_2ESUC}\ V1n))))) \tag{21}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ (p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \text{c\_2Enum\_2ESUC}\ V0n))\ (ap \\ \text{c\_2Enum\_2ESUC}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\ V0n) \\ V1m))))) \tag{22}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D} \\ \text{c\_2Enum\_2E0})\ V0n))) \tag{23}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D} \\ V0n)\ \text{c\_2Enum\_2E0}))\Leftrightarrow(V0n = \text{c\_2Enum\_2E0}))) \tag{24}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \text{c\_2Enum\_2E0})\ V0m) = \text{c\_2Enum\_2E0})\land \\ (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \text{c\_2Enum\_2E0}) = \text{c\_2Enum\_2E0})\land \\ (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL} \\ (ap\ \text{c\_2Earithmetic\_2EBIT1}\ \text{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m)\land \\ (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL} \\ (ap\ \text{c\_2Earithmetic\_2EBIT1}\ \text{c\_2Earithmetic\_2EZERO}))) = V0m)\land(\\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \text{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap \\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)) \\ V1n))\land((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \text{c\_2Enum\_2ESUC}\ V1n)) = \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\ V0m)\ V1n)))))))))))$$

$$\tag{25}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ \forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D} \\ V0m)\ V1n))\land(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ (\\ ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{26}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((p \; (ap \; (ap \; \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap \; (ap \; \mathsf{c\_2Earithmetic\_2E\_2B} \; V0m) \; V1n)) \; (ap \; (ap \; \mathsf{c\_2Earithmetic\_2E\_2B}$$
$$V0m) \; V2p)))\Leftrightarrow(p \; (ap \; (ap \; \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \; V1n) \; V2p))))))) \tag{27}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(\neg(p \; (ap \; (ap \; \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \; V0m) \; V1n)))\Leftrightarrow(p \; (ap \; (ap \; \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap \; \mathsf{c\_2Enum\_2ESUC} \; V1n)) \; V0m))))) \tag{28}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap \; \mathsf{c\_2Enum\_2ESUC} \; V0n) = (ap \; (ap$$
$$c\_2Earithmetic\_2E\_2B \; (ap \; \mathsf{c\_2Earithmetic\_2ENUMERAL} \; (ap \; \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO}))) \; V0n))) \tag{29}$$

Assume the following.

$$True \tag{30}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \; V0t1)\Rightarrow(p \; V1t2))\Rightarrow(((p$$
$$V1t2)\Rightarrow(p \; V0t1))\Rightarrow((p \; V0t1)\Leftrightarrow(p \; V1t2)))))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p \; V0t))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty \; A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p \; V0t))\Leftrightarrow(p \; V0t))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p \; V0t))\Leftrightarrow(p \; V0t))\wedge((((p \; V0t)\wedge True)\Leftrightarrow$$
$$(p \; V0t))\wedge(((False\wedge(p \; V0t))\Leftrightarrow False)\wedge((((p \; V0t)\wedge False)\Leftrightarrow False)\wedge$$
$$(((p \; V0t)\wedge(p \; V0t))\Leftrightarrow(p \; V0t)))))))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p \; V0t))\Leftrightarrow(p \; V0t))\wedge((((p \; V0t)\Rightarrow True)\Leftrightarrow$$
$$True)\wedge(((False\Rightarrow(p \; V0t))\Leftrightarrow True)\wedge((((p \; V0t)\Rightarrow(p \; V0t))\Leftrightarrow True)\wedge(($$
$$(p \; V0t)\Rightarrow False)\Leftrightarrow(\neg(p \; V0t))))))))) \tag{35}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \; V0t)))\Leftrightarrow(p \; V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge$$
$$((\neg False)\Leftrightarrow True))) \tag{36}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{37}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (p\ V1Q))))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{40}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{41}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Ellist\_2Ellist\ A\_27a).((V0l = (\text{c\_2Ellist\_2ELNIL}\ A\_27a)) \vee (\exists V1h \in A\_27a.(\exists V2t \in (ty\_2Ellist\_2Ellist\ A\_27a).(V0l = (ap\ (ap\ (\text{c\_2Ellist\_2ELCONS}\ A\_27a)\ V1h)\ V2t)))))) \tag{44}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow ((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ellist\_2ELNTH$$
$$A\_27a)\ V0n)\ (\textsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\textsf{c\_2Eoption\_2ENONE}\ A\_27a))) \wedge$$
$$((\forall V1h \in A\_27b.(\forall V2t \in (ty\_2Ellist\_2Ellist\ A\_27b).$$
$$((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27b)\ \textsf{c\_2Enum\_2E0})\ (ap\ (ap\ (\textsf{c\_2Ellist\_2ELCONS}$$
$$A\_27b)\ V1h)\ V2t)) = (ap\ (\textsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1h)))) \wedge (\forall V3n \in$$
$$ty\_2Enum\_2Enum.(\forall V4h \in A\_27c.(\forall V5t \in (ty\_2Ellist\_2Ellist$$
$$A\_27c).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27c)\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V3n))$$
$$(ap\ (ap\ (\textsf{c\_2Ellist\_2ELCONS}\ A\_27c)\ V4h)\ V5t)) = (ap\ (ap\ (c\_2Ellist\_2ELNTH$$
$$A\_27c)\ V3n)\ V5t)))))))$$

$$(45)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \textsf{c\_2Enum\_2E0})) \wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \textsf{c\_2Enum\_2ESUC}$$
$$V1n))))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))$$

$$(46)$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$\text{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \text{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V3m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Enumeral\_2EiZ}\ (ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \text{c\_2Enum\_2E0})\ V4n) = \text{c\_2Enum\_2E0}))\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)\ \text{c\_2Enum\_2E0}) = \text{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
$ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V7m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$\text{c\_2Enum\_2E0})\ V8n) = \text{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \text{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V11m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
$\text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}$
$V12n))) = \text{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP\ \text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \text{c\_2Earithmetic\_2EBIT2}\ V13n))) = \text{c\_2Enum\_2E0}))\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \text{c\_2Enum\_2E0}) =$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}\ \text{c\_2Earithmetic\_2EZERO}))))\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V15n))$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \text{c\_2Enum\_2ESUC}$
$\text{c\_2Enum\_2E0}) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}$
$\text{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap\ \text{c\_2Enum\_2ESUC}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \text{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \text{c\_2Eprim\_\_rec\_2EPRE}\ \text{c\_2Enum\_2E0}) =$
$\text{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \text{c\_2Eprim\_\_rec\_2EPRE}$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \text{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \text{c\_2Enum\_2E0})\Leftrightarrow(V19n = \text{c\_2Earithmetic\_2EZERO})))\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.((\text{c\_2Enum\_2E0} = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V20n))\Leftrightarrow(V20n = \text{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V21n) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}$
$V23n)\ \text{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ \text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V24n)))\Leftrightarrow(p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ \text{c\_2Earithmetic\_2EZERO})$
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V25n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}$
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3E}$
$\text{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3E}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V28n))\ \text{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ \text{c\_2Earithmetic\_2EZERO})$
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3E}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V29n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}$
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}$
$\text{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\
((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ \mathsf{c\_2Earithmetic\_2EZERO}) \ V0n))\Leftrightarrow\\
True)\wedge(((p \ (ap \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}\\
V0n)) \ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
(ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V0n)) \ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge\\
(((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}\\
V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m)))\Leftrightarrow(p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
V0n) \ V1m)))\wedge(((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}\\
V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m)))\Leftrightarrow(p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
V0n) \ V1m)))\wedge(((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}\\
V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m)))\Leftrightarrow(\neg(p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
V1m) \ V0n))))\wedge((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}\\
V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m)))\Leftrightarrow(p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
V0n) \ V1m)))))))))))$$

$$(48)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in\\
A\_27a.(((ap \ (\mathsf{c\_2Eoption\_2ESOME} \ A\_27a) \ V0x) = (ap \ (\mathsf{c\_2Eoption\_2ESOME}\\
A\_27a) \ V1y))\Leftrightarrow(V0x = V1y))))$$

$$(49)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\\
A\_27a) = (ap \ (\mathsf{c\_2Eoption\_2ESOME} \ A\_27a) \ V0x))))$$

$$(50)$$

Theorem 1

$$\forall A\_27a.nonempty \ A\_27a\Rightarrow(\forall V0ll \in (ty\_2Ellist\_2Ellist\\
A\_27a).(\forall V1x \in A\_27a.(\forall V2n1 \in ty\_2Enum\_2Enum.(\forall V3n2 \in\\
ty\_2Enum\_2Enum.((((ap \ (ap \ (c\_2Ellist\_2ELNTH \ A\_27a) \ V2n1) \ V0ll) =\\
(ap \ (\mathsf{c\_2Eoption\_2ESOME} \ A\_27a) \ V1x))\wedge(p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
V3n2) \ V2n1)))\Rightarrow(\exists V4y \in A\_27a.((ap \ (ap \ (c\_2Ellist\_2ELNTH\\
A\_27a) \ V3n2) \ V0ll) = (ap \ (\mathsf{c\_2Eoption\_2ESOME} \ A\_27a) \ V4y)))))))))$$