

thm_2Elogroot_2EEXP__MUL
(TMNvHbrPGGRiCEujvSj92j622yTGkjtdhqZ)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_$

Definition 8 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_Earithmetic_E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Definition 9 We define $c_Emin_E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_Ebool_E_3F$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 11 We define $c_Ebool_E_21$ to be $(ap (c_Ebool_E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 12 We define $c_Emin_E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21) 2) (\lambda V2t \in$

Definition 14 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21) 2) (\lambda V2t \in$

Definition 15 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_E_3D_3D_3E V0t) c_Ebool_E_21$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum.((ap (ap c_Earithmetic_EEXP \\ & V0m) c_Enum_2E0) = (ap c_Earithmetic_ENUMERAL (ap c_Earithmetic_EBIT1 \\ & c_Earithmetic_EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in \\ & ty_2Enum_2Enum.((ap (ap c_Earithmetic_EEXP V1m) (ap c_Enum_2ESUC \\ & V2n)) = (ap (ap c_Earithmetic_E_2A V1m) (ap (ap c_Earithmetic_EEXP \\ & V1m) V2n)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap (ap c_Earithmetic_E_2A c_Enum_2E0) V0m) = c_Enum_2E0) \wedge \\ & (((ap (ap c_Earithmetic_E_2A V0m) c_Enum_2E0) = c_Enum_2E0) \wedge \\ & (((ap (ap c_Earithmetic_E_2A (ap c_Earithmetic_ENUMERAL \\ & (ap c_Earithmetic_EBIT1 c_Earithmetic_EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_Earithmetic_E_2A V0m) (ap c_Earithmetic_ENUMERAL \\ & (ap c_Earithmetic_EBIT1 c_Earithmetic_EZERO))) = V0m) \wedge (\\ & ((ap (ap c_Earithmetic_E_2A (ap c_Enum_2ESUC V0m)) V1n) = (ap \\ & (ap c_Earithmetic_E_2B (ap (ap c_Earithmetic_E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_Earithmetic_E_2A V0m) (ap c_Enum_2ESUC V1n)) = \\ & (ap (ap c_Earithmetic_E_2B V0m) (ap (ap c_Earithmetic_E_2A \\ & V0m) V1n))))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Enum_2Enum. (\forall V1q \in ty_2Enum_2Enum. (\\
& \quad \forall V2n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP V2n) \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0p) V1q)) = (ap (ap c_2Earithmetic_2E_2A \\
& \quad (ap (ap c_2Earithmetic_2EEXP V2n) V0p)) (ap (ap c_2Earithmetic_2EEXP \\
& \quad \quad V2n) V1q))))))
\end{aligned} \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{13}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& \quad A_27a. (p V0t)) \Leftrightarrow (p V0t)))
\end{aligned} \tag{16}$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& \quad (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (28)$$

Assume the following.

$$(\forall V0P \in (2^{ty.2Enum.2Enum}).(((p (ap V0P c.2Enum.2E0)) \wedge (\forall V1n \in ty.2Enum.2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c.2Enum.2ESUC V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum.(p (ap V0P V2n)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Leftrightarrow (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \vee (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Rightarrow (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\neg(p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \quad (38)$$

Theorem 1

$$(\forall V0a \in ty_2Enum_2Enum.(\forall V1b \in ty_2Enum_2Enum.(\forall V2c \in ty_2Enum_2Enum.((ap \ (ap \ c_2Earithmetic_2EEXP \ (ap \ (ap \ c_2Earithmetic_2EEXP \ V0a) \ V1b)) \ V2c) = (ap \ (ap \ c_2Earithmetic_2EEXP \ V0a) \ (ap \ (ap \ c_2Earithmetic_2E_2A \ V1b) \ V2c))))))$$