# thm_2Emachine_ieee_2Efloat_fp32_nchotomy (TMY3LdupKoaPnJRs9x8SxfPd1rzLmEZnzH2)

October 26, 2020

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart \\ A0\ A1) \tag{1}$$

**Definition 1** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$
*of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$
*of type* $\iota \Rightarrow \iota$.

**Definition 3** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 $A$

**Definition 4** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{2}$$

Let $ty\_2Efcp\_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Ebit0\ A0) \tag{3}$$

Let $ty\_2Efcp\_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Ebit1\ A0) \tag{4}$$

Let $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_ieee\_2Efloat \\ A0\ A1) \tag{5}$$

Let $c\_2Ebinary\_ieee\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand \\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{6}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent$$
$$A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})$$
$$\tag{7}$$

Let $ty\_2Esum\_2Esum : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Esum\_2Esum$$
$$A0\ A1)$$
$$\tag{8}$$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{9}$$

Let $ty\_2Ebool\_2Eitself : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{10}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ebool\_2Ethe\_\_value\ A\_27a \in ($$
$$ty\_2Ebool\_2Eitself\ A\_27a)$$
$$\tag{11}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{12}$$

Let $c\_2Efcp\_2Edimindex : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{13}$$

**Definition 5** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($ c_2Emin_2E_3D $(2^{A\_27a}$

**Definition 6** *We define* c_2Ebool_2EF *to be* $(ap\ ($ c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

**Definition 7** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P{\Rightarrow}p\ Q)$
*of type* $\iota$.

**Definition 8** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

**Definition 9** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($ c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{15}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{16}$$

**Definition 10** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

**Definition 11** *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

**Definition 12** *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ c\_2Ebool\_2E\_2F\_5C$

**Definition 13** *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Emin\_2E\_40\ (A\_27a^{ty\_2Enum\_2Enu}$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart$$
$$A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)})$$
$$(17)$$

**Definition 14** *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (18)$$

**Definition 15** *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

**Definition 16** *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(19)$$

**Definition 17** *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 18** *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(20)$$

**Definition 19** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 20** *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebo$

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum})$$
$$(21)$$

**Definition 21** *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap$

**Definition 22** *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

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Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(22)

**Definition 23** *We define* c\_2Ebit\_2EDIV\_\_2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(23)

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(24)

**Definition 24** *We define* c\_2Ebit\_2EMOD\_\_2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

**Definition 25** *We define* c\_2Ebit\_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 26** *We define* c\_2Ebit\_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 27** *We define* c\_2Efcp\_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

**Definition 28** *We define* c\_2Ewords\_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c\_2Efcp\_2EFC

**Definition 29** *We define* c\_2Ewords\_2Ew2w *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a$

**Definition 30** *We define* c\_2Ebool\_2E\_5C\_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c\_2Ebool\_2E\_21 2) $(\lambda V2t \in$

**Definition 31** *We define* c\_2Earithmetic\_2E\_3C\_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

**Definition 32** *We define* c\_2Ewords\_2Eword\_\_lsl *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

**Definition 33** *We define* c\_2Ewords\_2Eword\_\_or *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1v$

**Definition 34** *We define* c\_2Ebool\_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

**Definition 35** *We define* c\_2Ewords\_2Eword\_\_join *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2$

**Definition 36** *We define* c\_2Ewords\_2Eword\_\_concat *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0v \in (ty\_2Ef$

Let $c\_2Ebinary\_ieee\_2Efloat\_\_Sign : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_ieee\_2Efloat\_\_Sign$$
$$A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})$$
(25)

**Definition 37** *We define* c\_2Emachine\_ieee\_2Efloat\_\_to\_\_fp32 *to be* $\lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ (ty$

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Let $c\_2Ebool\_2EARB : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \qquad (26)$$

**Definition 38** *We define* c_2Earithmetic_2EMIN *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 39** *We define* c_2Ewords_2Eword__bits *to be* $\lambda A\_27a : \iota.\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2$

**Definition 40** *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

**Definition 41** *We define* c_2Ewords_2Eword__extract *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0h \in ty\_2Enum\_2Enum$

**Definition 42** *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd : \iota\Rightarrow\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27u.nonempty\ A\_27u\Rightarrow\forall A\_27u$$
$$nonempty\ A\_27w\Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_f$$
$$A\_27t\ A\_27u\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27u\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})(\qquad (27)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd : \iota\Rightarrow\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow\forall A\_27x.$$
$$nonempty\ A\_27x\Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27$$
$$A\_27w\ A\_27x \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27x)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})((ty\_2Efl\qquad (28)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow c\_2Ebinary\_\_ieee\_$$
$$A\_27t\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})((ty\_2Efl\qquad (29)$$

**Definition 43** *We define* c_2Emachine__ieee_2Efp32__to__float *to be* $\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ (ty\_2Efcp$

Assume the following.
$$True \qquad (30)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow \qquad (31)$$
$$True))$$

Assume the following.

$$(\forall V0x \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit1\ ($$
$$ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))))$$
$$(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))).$$

$$((ap\ c\_2Emachine\_\_ieee\_2Efp32\_\_to\_\_float\ (ap\ c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp32$$
$$V0x)) = V0x))$$
$$(32)$$

**Theorem 1**

$(\forall V0x \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit1\ ($
$ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))))$
$(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))).$
$(\exists V1y \in (ty\_2Efcp\_2Ecart\ 2\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0$
$(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))))).$
$(V0x = (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efp32\_\_to\_\_float}\ V1y))))$