

thm_2Emachine_ieee_2Efloat__to__fp16__fp16__to__float (TMHxKuGV5GTMx5KKwGH6xzKAzDXWYo3dFb5)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ecombin_2EK` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 3 We define `c_2Ecombin_2ES` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define `c_2Ecombin_2EI` to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 5 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define `c_2Epred__set_2EUNIV` to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 7 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (2^{A_27a})) (2^{A_27a})))$

Definition 10 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 11 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EABS__prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS__prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (3)$$

Definition 13 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Ebool_2EF$

Definition 14 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 15 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 16 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E_21\ 2)$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (4)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (5)$$

Let $ty_2Efcf_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcf_2Ecart\ A0\ A1) \quad (6)$$

Let $c_2Ebinary_ieee_2Efloat_Significand_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand_fupd\ A_27t\ A_27u\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27u\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (7)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x.nonempty\ A_27x \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A_27t\ A_27w\ A_27x \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (8)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (9)$$

Let $c_2Ebinary_ieee_2Efloat_Sign_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign_fupd\ A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)} \quad (10)$$

Let $ty_2Efc_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Ebit0\ A0) \quad (11)$$

Let $ty_2Efc_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Ebit1\ A0) \quad (12)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 19 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 20 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 21 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 22 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (19)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (20)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (21)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (22)$$

Definition 23 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 24 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p\ x)\ \text{of type } \iota \Rightarrow \iota.$

Definition 25 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ A_27a)\ P)))$

Definition 26 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 27 We define $c_2Ebool_2E3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E2F_5C\ A_27a)\ P)))$

Definition 28 We define $c_2Efc_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E40\ (A_27a\ ty_2Enum_2Enum)))$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart\ A_27a\ A_27b \in ((A_27a\ (ty_2Efc_2Efinite_image\ A_27b))\ (ty_2Efc_2Ecart\ A_27a\ A_27b)) \quad (23)$$

Definition 29 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ A_27a\ A_27b)$

Let $c_2Earithmetic_2E2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})\ ty_2Enum_2Enum) \quad (24)$$

Definition 30 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (ap\ c_2Ebool_2E3F_21\ A_27a)\ t1)\ t2))))$

Definition 31 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 32 We define $c_2Earithmetic_2E3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 33 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a\ ty_2Enum_2Enum).(ap\ (ap\ (c_2Efc_2Edest_cart\ A_27a\ A_27b)\ g))))$

Definition 34 We define $c_Ewords_Eword_bits$ to be $\lambda A_27a : \iota.\lambda V0h \in ty_Eenum_Eenum.\lambda V1l \in ty_E$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (25)$$

Definition 35 We define c_Ebit_ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_Eenum_Eenum.(ap (ap (ap (c_Ebo$

Let $c_Esum_num_ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_ESUM \in ((ty_Eenum_Eenum^{(ty_Eenum_Eenum^{ty_Eenum_Eenum})})^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (26)$$

Definition 36 We define c_Ewords_Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efcf_Ecart\ 2\ A_27a).(ap (ap$

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (27)$$

Definition 37 We define $c_Ebit_EDIV_EXP$ to be $\lambda V0x \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum$

Let $c_Earithmetic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_EMOD \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (28)$$

Definition 38 We define $c_Ebit_EMOD_EXP$ to be $\lambda V0x \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum$

Definition 39 We define c_Ebit_EBITS to be $\lambda V0h \in ty_Eenum_Eenum.\lambda V1l \in ty_Eenum_Eenum.\lambda V$

Definition 40 We define c_Ebit_EBIT to be $\lambda V0b \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum.(ap$

Definition 41 We define c_Ewords_En2w to be $\lambda A_27a : \iota.\lambda V0n \in ty_Eenum_Eenum.(ap (c_Efcf_EFC$

Definition 42 We define c_Ewords_Ew2w to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w \in (ty_Efcf_Ecart\ 2\ A_27a$

Definition 43 We define $c_Ecombin_Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Definition 44 We define $c_Ewords_Eword_extract$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0h \in ty_Eenum_Eenum$

Definition 45 We define $c_Emachine_ieee_Efp16_to_float$ to be $\lambda V0w \in (ty_Efcf_Ecart\ 2\ (ty_Efcf$

Let $c_Ebinary_ieee_Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_Significand\ A_27t\ A_27w \in ((ty_Efcf_Ecart\ 2\ A_27t)^{(ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)}) \quad (29)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (30)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (31)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (32)$$

Definition 46 We define $c_2Ewords_2Eword_lsl$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1x \in (ty_2Eone_2Eone)^{A_27a}.$

Definition 47 We define $c_2Ewords_2Eword_or$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1x \in (ty_2Eone_2Eone)^{A_27a}.$

Definition 48 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b.f\ x))$

Definition 49 We define $c_2Ewords_2Eword_join$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).$

Definition 50 We define $c_2Ewords_2Eword_concat$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).$

Definition 51 We define $c_2Emachine_ieee_2Efloat_to_fp16$ to be $\lambda V0x \in (ty_2Ebinary_ieee_2Efloat\ (ty_2Eone_2Eone)^{A_27a}).$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Eenum_2Eenum}) \quad (33)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Eenum_2Eenum}) \quad (34)$$

Definition 52 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum.$

Definition 53 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum.$

Definition 54 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Eenum_2Eenum.(ap\ (ap\ (ap\ (c_2Ebool_2ELET\ m))))$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Eenum_2Eenum)^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum} \quad (35)$$

Definition 55 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Eenum_2Eenum.(ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2E_2A\ n))$

Definition 56 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Enumeral_2EiSUB : \iota$ be given. Assume the following.

$$c_2Enumeral_2EiSUB \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^2) \quad (36)$$

Definition 57 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B V0m)$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m)) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (38)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (39)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D V0m) c_2Enum_2E0) = V0m))) \quad (40)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))))))) \quad (41)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V0m))) \quad (43)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V0n) = (ap (ap c_2Earithmetic_2E_2B V0n) V0n))) \quad (44)$$

Assume the following.

$$(\forall V0c \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D V0c) V0c) = c_2Enum_2E0)) \quad (45)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p)))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \quad (46)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m)))))) \quad (47)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0n))) \quad (48)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p)))))) \quad (49)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EMIN V0n) V0n) = V0n)) \quad (50)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27t}.nonempty\ A_{.27t} \Rightarrow \forall A_{.27u}.nonempty\ A_{.27u} \Rightarrow \forall A_{.27w}. \\
& \quad nonempty\ A_{.27w} \Rightarrow \forall A_{.27x}.nonempty\ A_{.27x} \Rightarrow ((\forall V0f0 \in \\
& \quad ((ty_2EfcP_2Ecart\ 2\ A_{.27x})^{(ty_2EfcP_2Ecart\ 2\ A_{.27w})}).(\forall V1f \in \\
& \quad (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27x})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent_fupd \\
& \quad A_{.27t}\ A_{.27w}\ A_{.27x})\ V0f0)\ V1f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V1f)))) \wedge ((\forall V2f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27u})^{(ty_2EfcP_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V3f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Sign\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand_fupd \\
& \quad A_{.27t}\ A_{.27u}\ A_{.27w})\ V2f0)\ V3f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V3f)))) \wedge ((\forall V4f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)}. \\
& \quad (\forall V5f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign_fupd \\
& \quad A_{.27t}\ A_{.27w})\ V4f0)\ V5f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V5f)))) \wedge ((\forall V6f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27u})^{(ty_2EfcP_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V7f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand_fupd \\
& \quad A_{.27t}\ A_{.27u}\ A_{.27w})\ V6f0)\ V7f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V7f)))) \wedge ((\forall V8f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)}. \\
& \quad (\forall V9f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Sign_fupd\ A_{.27t}\ A_{.27w})\ V8f0)\ V9f)) = \\
& \quad (ap\ (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t}\ A_{.27w})\ V9f)))) \wedge \\
& \quad ((\forall V10f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27x})^{(ty_2EfcP_2Ecart\ 2\ A_{.27w})}). \\
& \quad (\forall V11f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ \\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t}\ A_{.27x})\ (ap\ (ap \\
& \quad (c_2EbinaRy_ieee_2Efloat_Exponent_fupd\ A_{.27t}\ A_{.27w}\ A_{.27x}) \\
& \quad V10f0)\ V11f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t} \\
& \quad A_{.27w})\ V11f)))) \wedge ((\forall V12f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)}. \\
& \quad (\forall V13f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ \\
& \quad (c_2EbinaRy_ieee_2Efloat_Sign\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign_fupd \\
& \quad A_{.27t}\ A_{.27w})\ V12f0)\ V13f)) = (ap\ V12f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V13f)))) \wedge ((\forall V14f0 \in ((ty_2EfcP_2Ecart\ 2 \\
& \quad A_{.27x})^{(ty_2EfcP_2Ecart\ 2\ A_{.27w})}).(\forall V15f \in (ty_2EbinaRy_ieee_2Efloat \\
& \quad A_{.27t}\ A_{.27w}).((ap\ (c_2EbinaRy_ieee_2Efloat_Exponent\ A_{.27t} \\
& \quad A_{.27x})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent_fupd\ A_{.27t} \\
& \quad A_{.27w}\ A_{.27x})\ V14f0)\ V15f)) = (ap\ V14f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V15f)))) \wedge ((\forall V16f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27u})^{(ty_2EfcP_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V17f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ \\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27u}\ A_{.27w})\ (ap\ (ap \\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand_fupd\ A_{.27t}\ A_{.27u}\ A_{.27w}) \\
& \quad V16f0)\ V17f)) = (ap\ V16f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand \\
& \quad A_{.27t}\ A_{.27w})\ V17f)))))))))))))
\end{aligned}$$

(51)

Assume the following.

$$True \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in A_27a. ((ap\ (ap\ (c_2Ebool_2ELET \\ A_27a\ A_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ p\ V0t)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (60)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (61)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (62)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27b.((ap (ap (c.2Ecombin_2EK A.27a A.27b) V0x) V1y) = V0x))) \quad (63)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((ap (c.2Ecombin_2EI A.27a) V0x) = V0x)) \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ & (ap (c.2EfcP_2Edimindex (ty_2Esum_2Esum A.27a A.27b)) (c.2Ebool_2Ethe_value \\ & (ty_2Esum_2Esum A.27a A.27b))) = (ap (ap (ap (c.2Ebool_2ECOND ty_2Enum_2Enum) \\ & (ap (ap c.2Ebool_2E_2F_5C (ap (c.2Epred_set_2EFINITE A.27a) \\ & (c.2Epred_set_2EUNIV A.27a))) (ap (c.2Epred_set_2EFINITE \\ & A.27b) (c.2Epred_set_2EUNIV A.27b)))) (ap (ap c.2Earithmetic_2E_2B \\ & (ap (c.2EfcP_2Edimindex A.27a) (c.2Ebool_2Ethe_value A.27a))) \\ & (ap (c.2EfcP_2Edimindex A.27b) (c.2Ebool_2Ethe_value A.27b)))) \\ & (ap c.2Earithmetic_2ENUMERAL (ap c.2Earithmetic_2EBIT1 c.2Earithmetic_2EZERO)))) \quad (65) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((ap (c.2EfcP_2Edimindex (ty_2EfcP_2Ebit0 \\ & A.27a)) (c.2Ebool_2Ethe_value (ty_2EfcP_2Ebit0 A.27a))) = (\\ & ap (ap (ap (c.2Ebool_2ECOND ty_2Enum_2Enum) (ap (c.2Epred_set_2EFINITE \\ & A.27a) (c.2Epred_set_2EUNIV A.27a))) (ap (ap c.2Earithmetic_2E_2A \\ & (ap c.2Earithmetic_2ENUMERAL (ap c.2Earithmetic_2EBIT2 c.2Earithmetic_2EZERO)))) \\ & (ap (c.2EfcP_2Edimindex A.27a) (c.2Ebool_2Ethe_value A.27a)))) \\ & (ap c.2Earithmetic_2ENUMERAL (ap c.2Earithmetic_2EBIT1 c.2Earithmetic_2EZERO)))) \quad (66) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((p (ap (c.2Epred_set_2EFINITE \\ & (ty_2EfcP_2Ebit0 A.27a)) (c.2Epred_set_2EUNIV (ty_2EfcP_2Ebit0 \\ & A.27a)))) \Leftrightarrow (p (ap (c.2Epred_set_2EFINITE A.27a) (c.2Epred_set_2EUNIV \\ & A.27a)))) \quad (67) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\text{ap } (c_2Efc_2Edimindex \ (ty_2Efc_2Ebit1 \\
& \quad A_{27a})) \ (c_2Ebool_2Ethe_value \ (ty_2Efc_2Ebit1 \ A_{27a}))) = (\\
& \text{ap } (\text{ap } (\text{ap } (c_2Ebool_2ECOND \ ty_2Enum_2Enum) \ (\text{ap } (c_2Epred_set_2EFINITE \\
& \quad A_{27a}) \ (c_2Epred_set_2EUNIV \ A_{27a}))) \ (\text{ap } (\text{ap } c_2Earithmetic_2E_2B \\
& \quad (\text{ap } (\text{ap } c_2Earithmetic_2E_2A \ (\text{ap } c_2Earithmetic_2ENUMERAL \ (\text{ap} \\
& \quad c_2Earithmetic_2EBIT2 \ c_2Earithmetic_2EZERO)))) \ (\text{ap } (c_2Efc_2Edimindex \\
& \quad A_{27a}) \ (c_2Ebool_2Ethe_value \ A_{27a})))) \ (\text{ap } c_2Earithmetic_2ENUMERAL \\
& (\text{ap } c_2Earithmetic_2EBIT1 \ c_2Earithmetic_2EZERO)))) \ (\text{ap } c_2Earithmetic_2ENUMERAL \\
& \quad (\text{ap } c_2Earithmetic_2EBIT1 \ c_2Earithmetic_2EZERO))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((p \ (\text{ap } (c_2Epred_set_2EFINITE \\
& \quad (ty_2Efc_2Ebit1 \ A_{27a})) \ (c_2Epred_set_2EUNIV \ (ty_2Efc_2Ebit1 \\
& \quad A_{27a})))) \Leftrightarrow (p \ (\text{ap } (c_2Epred_set_2EFINITE \ A_{27a}) \ (c_2Epred_set_2EUNIV \\
& \quad A_{27a}))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& ((\text{ap } (c_2Efc_2Edimindex \ ty_2Eone_2Eone) \ (c_2Ebool_2Ethe_value \\
& \quad ty_2Eone_2Eone)) = (\text{ap } c_2Earithmetic_2ENUMERAL \ (\text{ap } c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (p \ (\text{ap } (c_2Epred_set_2EFINITE \ ty_2Eone_2Eone) \ (c_2Epred_set_2EUNIV \\
& \quad ty_2Eone_2Eone)))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (((\text{ap } c_2Enum_2ESUC \ c_2Earithmetic_2EZERO) = (\text{ap } c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((\text{ap} \\
& \quad c_2Enum_2ESUC \ (\text{ap } c_2Earithmetic_2EBIT1 \ V0n)) = (\text{ap } c_2Earithmetic_2EBIT2 \\
& \quad V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((\text{ap } c_2Enum_2ESUC \ (\text{ap } c_2Earithmetic_2EBIT2 \\
& \quad V1n)) = (\text{ap } c_2Earithmetic_2EBIT1 \ (\text{ap } c_2Enum_2ESUC \ V1n))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
 (\forall V0n \in ty_2Enum_2Enum. (((ap\ c_2Enumeral_2EiiSUC\ c_2Earithmetic_2EZERO) = \\
 (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
 (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT1\ (\\
 ap\ c_2Enum_2ESUC\ V0n))) \wedge ((ap\ c_2Enumeral_2EiiSUC\ (ap\ c_2Earithmetic_2EBIT2 \\
 V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ V0n))))))
 \end{aligned}
 \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1b \in 2. (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3m \in ty_2Enum_2Enum. (((ap (ap (ap c_2Enumeral_2EiSUB \\
& V1b) c_2Earithmetic_2EZERO) V0x) = c_2Earithmetic_2EZERO) \wedge (\\
& ((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) c_2Earithmetic_2EZERO) = \\
& V2n) \wedge (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) c_2Earithmetic_2EZERO) = (ap c_2Enumeral_2EiDUB V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge ((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))))))))))))))))) \\
& \hspace{15em} (78)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V0n) \\
& V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n)) (ap c_2Earithmetic_2ENUMERAL (ap (ap (ap c_2Enumeral_2EiSUB \\
& c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0)))) \\
& \hspace{15em} (79)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT2\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) \wedge ((ap\ c_2Enumeral_2EiDUB\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO)))) \\
& \hspace{15em} (80)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. (\\
& \quad ((ap\ (ap\ c_2Earithmetic_2EMIN\ c_2Enum_2E0)\ V0x) = c_2Enum_2E0) \wedge \\
& \quad (((ap\ (ap\ c_2Earithmetic_2EMIN\ V0x)\ c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& \quad ((ap\ (ap\ c_2Earithmetic_2EMIN\ (ap\ c_2Earithmetic_2ENUMERAL\ V0x)) \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ V1y)) = (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Enum_2Enum)\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& \quad \quad V0x)\ V1y))\ V0x)\ V1y)))))) \\
& \hspace{15em} (81)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty_2Enum_2Enum. ((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2ENUMERAL\ V0i)) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Enumeral_2EiDUB\ V0i)))) \\
& \hspace{15em} (82)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Efc_2Ecart \\
& \quad 2\ A_27a). (\forall V1b \in (ty_2Efc_2Ecart\ 2\ A_27a). ((ap\ (ap\ (c_2Ewords_2Eword_or \\
& \quad \quad A_27a)\ V0a)\ V1b) = (ap\ (ap\ (c_2Ewords_2Eword_or\ A_27a)\ V1b)\ V0a)))) \\
& \hspace{15em} (83)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0w \in (ty_2Efc_2Ecart\ 2\ A_27a). ((ap\ (c_2Ewords_2Ew2w \\
& \quad \quad A_27a\ A_27b)\ V0w) = (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract\ A_27a \\
& \quad \quad A_27b)\ (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2Efc_2Edimindex\ A_27a) \\
& \quad \quad (c_2Ebool_2Ethe_value\ A_27a)))\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ c_2Enum_2E0 \\
& \quad \quad V0w))) \\
& \hspace{15em} (84)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in ty_2Enum_2Enum. (\\
& \quad \forall V1l \in ty_2Enum_2Enum. (\forall V2w \in (ty_2Efc_2Ecart\ 2 \\
& \quad \quad A_27a). ((ap\ (ap\ (ap\ (c_2Ewords_2Eword_bits\ A_27a)\ V0h)\ V1l)\ V2w) = \\
& \quad \quad (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract\ A_27a\ A_27a)\ V0h)\ V1l)\ V2w)))))) \\
& \hspace{15em} (85)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0w \in (ty_2EfcP_2Ecart\ 2\ A_27c). (\forall V1h \in \\
& \quad ty_2Enum_2Enum. (\forall V2l \in ty_2Enum_2Enum. (\forall V3m \in ty_2Enum_2Enum. \\
& \quad (\forall V4n \in ty_2Enum_2Enum. ((ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract \\
& \quad A_27b\ A_27a)\ V1h)\ V2l)\ (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract\ A_27c \\
& \quad A_27b)\ V3m)\ V4n)\ V0w))) = (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract \\
& \quad A_27c\ A_27a)\ (ap\ (ap\ c_2Earithmetic_2EMIN\ V3m)\ (ap\ (ap\ c_2Earithmetic_2EMIN \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2B\ V1h)\ V4n))\ (ap\ (ap\ c_2Earithmetic_2EMIN \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2EfcP_2Edimindex\ A_27c)\ (\\
& \quad c_2Ebool_2Ethe_value\ A_27c)))\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ (ap\ (ap \\
& \quad c_2Earithmetic_2E_2D\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (c_2EfcP_2Edimindex \\
& \quad A_27b)\ (c_2Ebool_2Ethe_value\ A_27b)))\ V4n))\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ (ap \\
& \quad (ap\ c_2Earithmetic_2E_2B\ V2l)\ V4n))\ V0w))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2EfcP_2Ecart \\
& \quad 2\ A_27a). (\forall V1h \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2EfcP_2Edimindex\ A_27a)\ (\\
& \quad c_2Ebool_2Ethe_value\ A_27a)))\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V1h)) \Rightarrow \\
& \quad ((ap\ (ap\ (ap\ (c_2Ewords_2Eword_bits\ A_27a)\ V1h)\ c_2Enum_2E0) \\
& \quad V0w) = V0w)))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0h \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\forall V2m_27 \in \\
& \quad ty_2Enum_2Enum. (\forall V3l \in ty_2Enum_2Enum. (\forall V4s \in ty_2Enum_2Enum. \\
& \quad (\forall V5w \in (ty_2Efc_2Ecart\ 2\ A_27a). (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& \quad V3l)\ V1m)) \wedge ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V2m_27)\ V0h)) \wedge \\
& \quad ((V2m_27 = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1m)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \wedge (V4s = \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2D\ V2m_27)\ V3l)))))) \Rightarrow ((ap\ (ap\ (c_2Ewords_2Eword_or \\
& \quad A_27b)\ (ap\ (ap\ (c_2Ewords_2Eword_lsl\ A_27b)\ (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract \\
& \quad A_27a\ A_27b)\ V0h)\ V2m_27)\ V5w))\ V4s))\ (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract \\
& \quad A_27a\ A_27b)\ V1m)\ V3l)\ V5w)) = (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ c_2Earithmetic_2EMIN\ V0h)\ (ap\ (ap\ c_2Earithmetic_2EMIN \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap \\
& \quad (c_2Efc_2Edimindex\ A_27b)\ (c_2Ebool_2Ethe_value\ A_27b)))) \\
& \quad V3l))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))))\ (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (\\
& \quad c_2Efc_2Edimindex\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))\ (\\
& \quad ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \\
& \quad V3l)\ V5w)))))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& ((ap\ (c_2Efc_2Edimindex\ ty_2Eone_2Eone)\ (c_2Ebool_2Ethe_value \\
& \quad ty_2Eone_2Eone)) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& ((ap\ (c_2Efc_2Edimindex\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0 \\
& \quad ty_2Eone_2Eone)))\ (c_2Ebool_2Ethe_value\ (ty_2Efc_2Ebit1 \\
& \quad (ty_2Efc_2Ebit0\ ty_2Eone_2Eone)))) = (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& ((ap\ (c_2Efc_2Edimindex\ (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit1 \\
& \quad (ty_2Efc_2Ebit0\ ty_2Eone_2Eone))))\ (c_2Ebool_2Ethe_value \\
& \quad (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0\ ty_2Eone_2Eone)))))) = \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2 \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& ((ap (c_2Efc_2Edimindex (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) (c_2Ebool_2Ethe_value \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& ty_2Eone_2Eone)))))) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{92}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in (ty_2Efc_2Ecart\ 2 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))))).((ap \\
& c_2Emachine_iee_2Efloat_to_fp16 (ap c_2Emachine_iee_2Efp16_to_float \\
& V0x)) = V0x))
\end{aligned}$$