

# thm\_2Emachine\_ieee\_2Efloat\_\_to\_\_fp64\_\_fp64\_\_to\_\_float (TMdp3KkVHsKC71zgdtPzJTS41Nwy7yvRjmo)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ecombin_2EK` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

**Definition 3** We define `c_2Ecombin_2ES` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

**Definition 4** We define `c_2Ecombin_2EI` to be  $\lambda A_27a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A_27a (A_27a^{A_27a})) A_27a))$

**Definition 5** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 6** We define `c_2Epred__set_2EUNIV` to be  $\lambda A_27a : \iota. (\lambda V0x \in A_27a. \text{c_2Ebool_2ET})$ .

**Definition 7** We define `c_2Ebool_2EIN` to be  $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (\text{ap } V1f V0x)))$

**Definition 8** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a})) (2^{A_27a}))))$

**Definition 10** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

**Definition 11** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 A1) \quad (1)$$

Let `c_2Epair_2EABS__prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Epair_2EABS__prod } A_27a A_27b \in ((\text{ty_2Epair_2Eprod } A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b})}) \quad (3)$$

**Definition 13** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Ebool\_2EF$

**Definition 14** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 16** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 2)$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2EARB A\_27a \in A\_27a \quad (4)$$

Let  $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efloat A0 A1) \quad (5)$$

Let  $ty\_2Efcf\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcf\_2Ecart A0 A1) \quad (6)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27u.nonempty A\_27u \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd A\_27t A\_27u A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat A\_27u A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)}) \quad (7)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow \forall A\_27x.nonempty A\_27x \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd A\_27t A\_27w A\_27x \in (((ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27x)^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)})^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)}) \quad (8)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \quad (9)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign\_fupd : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign\_fupd\ A\_27t\ A\_27w \in (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)} \quad (10)$$

Let  $ty\_2Efc\_2Ebit0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efc\_2Ebit0\ A0) \quad (11)$$

Let  $ty\_2Efc\_2Ebit1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efc\_2Ebit1\ A0) \quad (12)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (13)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (14)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (15)$$

**Definition 17** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 18** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (17)$$

**Definition 19** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 20** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 21** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 22** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Efc\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efc\_2Efinite\_image\ A0) \quad (19)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (20)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (21)$$

Let  $c\_2Efc\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efc\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (22)$$

**Definition 23** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E7E))$

**Definition 24** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota.$

**Definition 25** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E40\ A\_27a\ P))))$

**Definition 26** We define  $c\_2Eprim\_rec\_2E3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 27** We define  $c\_2Ebool\_2E3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ c\_2Ebool\_2E2F\_5C\ A\_27a\ P))))$

**Definition 28** We define  $c\_2Efc\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Emin\_2E40\ (A\_27a^{ty\_2Enum\_2Enum})))$

Let  $c\_2Efc\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efc\_2Edest\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efc\_2Efinite\_image\ A\_27b)})^{(ty\_2Efc\_2Ecart\ A\_27a\ A\_27b)}) \quad (23)$$

**Definition 29** We define  $c\_2Efc\_2Efc\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efc\_2Ecart\ A\_27a\ A\_27b)$

Let  $c\_2Earithmetic\_2E2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (24)$$

**Definition 30** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ (c\_2Earithmetic\_2E2D\ t1\ t2))))))$

**Definition 31** We define  $c\_2Earithmetic\_2EMIN$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 32** We define  $c\_2Earithmetic\_2E3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 33** We define  $c\_2Efc\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap\ (c\_2Efc\_2Efc\_index\ A\_27a\ A\_27b\ g)))$

**Definition 34** We define  $c\_Ewords\_Eword\_bits$  to be  $\lambda A\_27a : \iota.\lambda V0h \in ty\_Eenum\_Eenum.\lambda V1l \in ty\_E$

Let  $c\_Earithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEXP \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum} \quad (25)$$

**Definition 35** We define  $c\_Ebit\_ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_Eenum\_Eenum.(ap (ap (ap (c\_Ebo$

Let  $c\_Esum\_num\_ESUM : \iota$  be given. Assume the following.

$$c\_Esum\_num\_ESUM \in ((ty\_Eenum\_Eenum^{(ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})})^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum} \quad (26)$$

**Definition 36** We define  $c\_Ewords\_Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_Efcf\_Ecart\ 2\ A\_27a).(ap (ap$

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum} \quad (27)$$

**Definition 37** We define  $c\_Ebit\_EDIV\_EXP$  to be  $\lambda V0x \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

Let  $c\_Earithmetic\_EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EMOD \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum} \quad (28)$$

**Definition 38** We define  $c\_Ebit\_EMOD\_EXP$  to be  $\lambda V0x \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

**Definition 39** We define  $c\_Ebit\_EBITS$  to be  $\lambda V0h \in ty\_Eenum\_Eenum.\lambda V1l \in ty\_Eenum\_Eenum.\lambda V$

**Definition 40** We define  $c\_Ebit\_EBIT$  to be  $\lambda V0b \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum.(ap$

**Definition 41** We define  $c\_Ewords\_En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_Eenum\_Eenum.(ap (c\_Efcf\_EFC$

**Definition 42** We define  $c\_Ewords\_Ew2w$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_Efcf\_Ecart\ 2\ A\_27a$

**Definition 43** We define  $c\_Ecombin\_Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

**Definition 44** We define  $c\_Ewords\_Eword\_extract$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0h \in ty\_Eenum\_Eenum$

**Definition 45** We define  $c\_Emachine\_ieee\_Efp64\_to\_float$  to be  $\lambda V0w \in (ty\_Efcf\_Ecart\ 2\ (ty\_Efcf$

Let  $c\_Ebinary\_ieee\_Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_Significand\ A\_27t\ A\_27w \in ((ty\_Efcf\_Ecart\ 2\ A\_27t)^{(ty\_Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)}) \quad (29)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (30)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (31)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (32)$$

**Definition 46** We define  $c\_2Ewords\_2Eword\_lsl$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).\lambda V1x \in (ty\_2Eone\_2Eone)^{A\_27a}.$

**Definition 47** We define  $c\_2Ewords\_2Eword\_or$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).\lambda V1x \in (ty\_2Eone\_2Eone)^{A\_27a}.$

**Definition 48** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27b.f\ x))$

**Definition 49** We define  $c\_2Ewords\_2Eword\_join$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).$

**Definition 50** We define  $c\_2Ewords\_2Eword\_concat$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0v \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).$

**Definition 51** We define  $c\_2Emachine\_ieee\_2Efloat\_to\_fp64$  to be  $\lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ (ty\_2Eone\_2Eone)^{A\_27a}).$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Eenum\_2Eenum}) \quad (33)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Eenum\_2Eenum}) \quad (34)$$

**Definition 52** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum.$

**Definition 53** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum.$

**Definition 54** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.(ap\ (ap\ (ap\ (c\_2Ebool\_2ELET\ m))))$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Eenum\_2Eenum)^{ty\_2Eenum\_2Eenum})^{ty\_2Eenum\_2Eenum} \quad (35)$$

**Definition 55** We define  $c\_2Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Eenum\_2Eenum.(ap\ c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2E\_2A\ n))$

**Definition 56** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Enumeral\_2EiSUB : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^2) \quad (36)$$

**Definition 57** We define  $c\_2Enumeral\_2EiDUB$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B V0m)$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \quad (38)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \quad (39)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap (ap c\_2Earithmetic\_2E\_2D \quad (40)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \quad (41)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \quad (42)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V0m))) \quad (43)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V0n) = (ap (ap c\_2Earithmetic\_2E\_2B V0n) V0n))) \quad (44)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D V0c) V0c) = c\_2Enum\_2E0)) \quad (45)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p)))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \quad (46)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \quad (47)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC V0n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0n))) \quad (48)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)))))) \quad (49)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EMIN V0n) V0n) = V0n)) \quad (50)$$



Assume the following.

$$\begin{aligned}
& \forall A_{.27t}.nonempty\ A_{.27t} \Rightarrow \forall A_{.27u}.nonempty\ A_{.27u} \Rightarrow \forall A_{.27w}. \\
& \quad nonempty\ A_{.27w} \Rightarrow \forall A_{.27x}.nonempty\ A_{.27x} \Rightarrow ((\forall V0f0 \in \\
& \quad ((ty\_2EfcP\_2Ecart\ 2\ A_{.27x})^{(ty\_2EfcP\_2Ecart\ 2\ A_{.27w})}).(\forall V1f \in \\
& \quad (ty\_2EbinaRy\_ieee\_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Sign \\
& \quad A_{.27t}\ A_{.27x})\ (ap\ (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Exponent\_fupd \\
& \quad A_{.27t}\ A_{.27w}\ A_{.27x})\ V0f0)\ V1f)) = (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V1f)))) \wedge ((\forall V2f0 \in ((ty\_2EfcP\_2Ecart\ 2\ A_{.27u})^{(ty\_2EfcP\_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V3f \in (ty\_2EbinaRy\_ieee\_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ ( \\
& \quad c\_2EbinaRy\_ieee\_2Efloat\_Sign\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Significand\_fupd \\
& \quad A_{.27t}\ A_{.27u}\ A_{.27w})\ V2f0)\ V3f)) = (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V3f)))) \wedge ((\forall V4f0 \in ((ty\_2EfcP\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2EfcP\_2Ecart\ 2\ ty\_2Eone\_2Eone)}. \\
& \quad (\forall V5f \in (ty\_2EbinaRy\_ieee\_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ ( \\
& \quad c\_2EbinaRy\_ieee\_2Efloat\_Exponent\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Sign\_fupd \\
& \quad A_{.27t}\ A_{.27w})\ V4f0)\ V5f)) = (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V5f)))) \wedge ((\forall V6f0 \in ((ty\_2EfcP\_2Ecart\ 2\ A_{.27u})^{(ty\_2EfcP\_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V7f \in (ty\_2EbinaRy\_ieee\_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ ( \\
& \quad c\_2EbinaRy\_ieee\_2Efloat\_Exponent\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Significand\_fupd \\
& \quad A_{.27t}\ A_{.27u}\ A_{.27w})\ V6f0)\ V7f)) = (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V7f)))) \wedge ((\forall V8f0 \in ((ty\_2EfcP\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2EfcP\_2Ecart\ 2\ ty\_2Eone\_2Eone)}. \\
& \quad (\forall V9f \in (ty\_2EbinaRy\_ieee\_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ ( \\
& \quad c\_2EbinaRy\_ieee\_2Efloat\_Significand\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ ( \\
& \quad c\_2EbinaRy\_ieee\_2Efloat\_Sign\_fupd\ A_{.27t}\ A_{.27w})\ V8f0)\ V9f)) = \\
& \quad (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Significand\ A_{.27t}\ A_{.27w})\ V9f)))) \wedge \\
& \quad ((\forall V10f0 \in ((ty\_2EfcP\_2Ecart\ 2\ A_{.27x})^{(ty\_2EfcP\_2Ecart\ 2\ A_{.27w})}). \\
& \quad (\forall V11f \in (ty\_2EbinaRy\_ieee\_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ \\
& \quad (c\_2EbinaRy\_ieee\_2Efloat\_Significand\ A_{.27t}\ A_{.27x})\ (ap\ (ap\ \\
& \quad (c\_2EbinaRy\_ieee\_2Efloat\_Exponent\_fupd\ A_{.27t}\ A_{.27w}\ A_{.27x}) \\
& \quad V10f0)\ V11f)) = (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Significand\ A_{.27t} \\
& \quad A_{.27w})\ V11f)))) \wedge ((\forall V12f0 \in ((ty\_2EfcP\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2EfcP\_2Ecart\ 2\ ty\_2Eone\_2Eone)}. \\
& \quad (\forall V13f \in (ty\_2EbinaRy\_ieee\_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ \\
& \quad (c\_2EbinaRy\_ieee\_2Efloat\_Sign\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Sign\_fupd \\
& \quad A_{.27t}\ A_{.27w})\ V12f0)\ V13f)) = (ap\ V12f0\ (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V13f)))) \wedge ((\forall V14f0 \in ((ty\_2EfcP\_2Ecart\ 2 \\
& \quad A_{.27x})^{(ty\_2EfcP\_2Ecart\ 2\ A_{.27w})}).(\forall V15f \in (ty\_2EbinaRy\_ieee\_2Efloat \\
& \quad A_{.27t}\ A_{.27w}).((ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Exponent\ A_{.27t} \\
& \quad A_{.27x})\ (ap\ (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Exponent\_fupd\ A_{.27t} \\
& \quad A_{.27w}\ A_{.27x})\ V14f0)\ V15f)) = (ap\ V14f0\ (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V15f)))) \wedge ((\forall V16f0 \in ((ty\_2EfcP\_2Ecart\ 2\ A_{.27u})^{(ty\_2EfcP\_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V17f \in (ty\_2EbinaRy\_ieee\_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ \\
& \quad (c\_2EbinaRy\_ieee\_2Efloat\_Significand\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ \\
& \quad (c\_2EbinaRy\_ieee\_2Efloat\_Significand\_fupd\ A_{.27t}\ A_{.27u}\ A_{.27w}) \\
& \quad V16f0)\ V17f)) = (ap\ V16f0\ (ap\ (c\_2EbinaRy\_ieee\_2Efloat\_Significand \\
& \quad A_{.27t}\ A_{.27w})\ V17f)))))))))))))
\end{aligned}$$

(51)

Assume the following.

$$True \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}). (\forall V1x \in A\_27a. ((ap\ (ap\ (c\_2Ebool\_2ELET \\ & A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ & A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (60)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (61)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (62)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27b.((ap (ap (c.2Ecombin_2EK A.27a A.27b) V0x) V1y) = V0x))) \quad (63)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((ap (c.2Ecombin_2EI A.27a) V0x) = V0x)) \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & (ap (c.2EfcP_2Edimindex (ty_2Esum_2Esum A.27a A.27b)) (c.2Ebool_2Ethe\_value \\ & (ty_2Esum_2Esum A.27a A.27b))) = (ap (ap (ap (c.2Ebool_2ECOND ty_2Enum_2Enum) \\ & (ap (ap c.2Ebool_2E_2F_5C (ap (c.2Epred\_set_2EFINITE A.27a) \\ & (c.2Epred\_set_2EUNIV A.27a))) (ap (c.2Epred\_set_2EFINITE \\ & A.27b) (c.2Epred\_set_2EUNIV A.27b)))) (ap (ap c.2Earithmetic_2E_2B \\ & (ap (c.2EfcP_2Edimindex A.27a) (c.2Ebool_2Ethe\_value A.27a))) \\ & (ap (c.2EfcP_2Edimindex A.27b) (c.2Ebool_2Ethe\_value A.27b)))) \\ & (ap c.2Earithmetic_2ENUMERAL (ap c.2Earithmetic_2EBIT1 c.2Earithmetic_2EZERO)))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((ap (c.2EfcP_2Edimindex (ty_2EfcP_2Ebit0 \\ & A.27a)) (c.2Ebool_2Ethe\_value (ty_2EfcP_2Ebit0 A.27a))) = ( \\ & ap (ap (ap (c.2Ebool_2ECOND ty_2Enum_2Enum) (ap (c.2Epred\_set_2EFINITE \\ & A.27a) (c.2Epred\_set_2EUNIV A.27a))) (ap (ap c.2Earithmetic_2E_2A \\ & (ap c.2Earithmetic_2ENUMERAL (ap c.2Earithmetic_2EBIT2 c.2Earithmetic_2EZERO)))) \\ & (ap (c.2EfcP_2Edimindex A.27a) (c.2Ebool_2Ethe\_value A.27a)))) \\ & (ap c.2Earithmetic_2ENUMERAL (ap c.2Earithmetic_2EBIT1 c.2Earithmetic_2EZERO)))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((p (ap (c.2Epred\_set_2EFINITE \\ & (ty_2EfcP_2Ebit0 A.27a)) (c.2Epred\_set_2EUNIV (ty_2EfcP_2Ebit0 \\ & A.27a)))) \Leftrightarrow (p (ap (c.2Epred\_set_2EFINITE A.27a) (c.2Epred\_set_2EUNIV \\ & A.27a)))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\text{ap } (c\_2Efc\_2Edimindex \ (ty\_2Efc\_2Ebit1 \\
& \quad A_{27a})) \ (c\_2Ebool\_2Ethe\_value \ (ty\_2Efc\_2Ebit1 \ A_{27a}))) = ( \\
& \text{ap } (\text{ap } (\text{ap } (c\_2Ebool\_2ECOND \ ty\_2Enum\_2Enum) \ (\text{ap } (c\_2Epred\_set\_2EFINITE \\
& \quad A_{27a}) \ (c\_2Epred\_set\_2EUNIV \ A_{27a}))) \ (\text{ap } (\text{ap } c\_2Earithmetic\_2E\_2B \\
& \quad (\text{ap } (\text{ap } c\_2Earithmetic\_2E\_2A \ (\text{ap } c\_2Earithmetic\_2ENUMERAL \ (\text{ap} \\
& \quad c\_2Earithmetic\_2EBIT2 \ c\_2Earithmetic\_2EZERO)))) \ (\text{ap } (c\_2Efc\_2Edimindex \\
& \quad A_{27a}) \ (c\_2Ebool\_2Ethe\_value \ A_{27a})))) \ (\text{ap } c\_2Earithmetic\_2ENUMERAL \\
& (\text{ap } c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO)))) \ (\text{ap } c\_2Earithmetic\_2ENUMERAL \\
& (\text{ap } c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((p \ (\text{ap } (c\_2Epred\_set\_2EFINITE \\
& \quad (ty\_2Efc\_2Ebit1 \ A_{27a})) \ (c\_2Epred\_set\_2EUNIV \ (ty\_2Efc\_2Ebit1 \\
& \quad A_{27a})))) \Leftrightarrow (p \ (\text{ap } (c\_2Epred\_set\_2EFINITE \ A_{27a}) \ (c\_2Epred\_set\_2EUNIV \\
& \quad A_{27a}))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& ((\text{ap } (c\_2Efc\_2Edimindex \ ty\_2Eone\_2Eone) \ (c\_2Ebool\_2Ethe\_value \\
& \quad ty\_2Eone\_2Eone)) = (\text{ap } c\_2Earithmetic\_2ENUMERAL \ (\text{ap } c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (p \ (\text{ap } (c\_2Epred\_set\_2EFINITE \ ty\_2Eone\_2Eone) \ (c\_2Epred\_set\_2EUNIV \\
& \quad ty\_2Eone\_2Eone)))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (((\text{ap } c\_2Enum\_2ESUC \ c\_2Earithmetic\_2EZERO) = (\text{ap } c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((\text{ap} \\
& \quad c\_2Enum\_2ESUC \ (\text{ap } c\_2Earithmetic\_2EBIT1 \ V0n)) = (\text{ap } c\_2Earithmetic\_2EBIT2 \\
& \quad V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((\text{ap } c\_2Enum\_2ESUC \ (\text{ap } c\_2Earithmetic\_2EBIT2 \\
& \quad V1n)) = (\text{ap } c\_2Earithmetic\_2EBIT1 \ (\text{ap } c\_2Enum\_2ESUC \ V1n))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
 (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ c\_2Enumeral\_2EiiSUC\ c\_2Earithmetic\_2EZERO) = \\
 (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
 (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT1\ ( \\
 ap\ c\_2Enum\_2ESUC\ V0n))) \wedge ((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ c\_2Earithmetic\_2EBIT2 \\
 V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ V0n))))))
 \end{aligned}
 \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) (ap c\_2Earithmetic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& (ap c\_2Earithmetic\_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V0n))) \wedge ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))))))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{77}$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1b \in 2. (\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3m \in ty\_2Enum\_2Enum. (((ap (ap (ap c\_2Enumeral\_2EiSUB \\
& V1b) c\_2Earithmic\_2EZERO) V0x) = c\_2Earithmic\_2EZERO) \wedge ( \\
& ((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) c\_2Earithmic\_2EZERO) = \\
& V2n) \wedge (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT1 \\
& V2n)) c\_2Earithmic\_2EZERO) = (ap c\_2Enumeral\_2EiDUB V2n)) \wedge \\
& (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmic\_2EBIT1 V3m)) = (ap c\_2Earithmic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmic\_2EBIT2 V3m)) = (ap c\_2Earithmic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT2 \\
& V2n)) c\_2Earithmic\_2EZERO) = (ap c\_2Earithmic\_2EBIT1 V2n)) \wedge \\
& (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmic\_2EBIT1 V3m)) = (ap c\_2Earithmic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge ((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmic\_2EBIT2 V3m)) = (ap c\_2Earithmic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))))))))))))))))) \\
& (78)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& (ap c\_2Earithmic\_2ENUMERAL (ap (ap c\_2Earithmic\_2E\_2D V0n) \\
& V1m)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V1m) V0n)) (ap c\_2Earithmic\_2ENUMERAL (ap (ap (ap c\_2Enumeral\_2EiSUB \\
& c\_2Ebool\_2ET) V0n) V1m))) c\_2Enum\_2E0)))) \\
& (79)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) \wedge ((ap\ c\_2Enumeral\_2EiDUB\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO)))) \\
& \hspace{15em} (80)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1y \in ty\_2Enum\_2Enum. ( \\
& \quad ((ap\ (ap\ c\_2Earithmetic\_2EMIN\ c\_2Enum\_2E0)\ V0x) = c\_2Enum\_2E0) \wedge \\
& \quad (((ap\ (ap\ c\_2Earithmetic\_2EMIN\ V0x)\ c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& \quad ((ap\ (ap\ c\_2Earithmetic\_2EMIN\ (ap\ c\_2Earithmetic\_2ENUMERAL\ V0x)) \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ V1y)) = (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Enum\_2Enum)\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& \quad \quad V0x)\ V1y))\ V0x)\ V1y)))))) \\
& \hspace{15em} (81)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty\_2Enum\_2Enum. ((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2ENUMERAL\ V0i)) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Enumeral\_2EiDUB\ V0i)))) \\
& \hspace{15em} (82)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2EfcP\_2Ecart \\
& \quad 2\ A\_27a). (\forall V1b \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a). ((ap\ (ap\ (c\_2Ewords\_2Eword\_or \\
& \quad \quad A\_27a)\ V0a)\ V1b) = (ap\ (ap\ (c\_2Ewords\_2Eword\_or\ A\_27a)\ V1b)\ V0a)))) \\
& \hspace{15em} (83)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a). ((ap\ (c\_2Ewords\_2Ew2w \\
& \quad \quad A\_27a\ A\_27b)\ V0w) = (ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_extract\ A\_27a \\
& \quad \quad A\_27b)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2EfcP\_2Edimindex\ A\_27a) \\
& \quad \quad (c\_2Ebool\_2Ethe\_value\ A\_27a)))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ c\_2Enum\_2E0 \\
& \quad \quad V0w))) \\
& \hspace{15em} (84)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V1l \in ty\_2Enum\_2Enum. (\forall V2w \in (ty\_2EfcP\_2Ecart\ 2 \\
& \quad \quad A\_27a). ((ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_bits\ A\_27a)\ V0h)\ V1l)\ V2w) = \\
& \quad \quad (ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_extract\ A\_27a\ A\_27a)\ V0h)\ V1l)\ V2w)))))) \\
& \hspace{15em} (85)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow (\forall V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27c). (\forall V1h \in \\
& ty\_2Enum\_2Enum. (\forall V2l \in ty\_2Enum\_2Enum. (\forall V3m \in ty\_2Enum\_2Enum. \\
& (\forall V4n \in ty\_2Enum\_2Enum. ((ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_extract \\
& A\_27b\ A\_27a)\ V1h)\ V2l)\ (ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_extract\ A\_27c \\
& A\_27b)\ V3m)\ V4n)\ V0w))) = (ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_extract \\
& A\_27c\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2EMIN\ V3m)\ (ap\ (ap\ c\_2Earithmetic\_2EMIN \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1h)\ V4n))\ (ap\ (ap\ c\_2Earithmetic\_2EMIN \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2EfcP\_2Edimindex\ A\_27c)\ ( \\
& c\_2Ebool\_2Ethe\_value\ A\_27c)))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ (ap\ (ap \\
& c\_2Earithmetic\_2E\_2D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (c\_2EfcP\_2Edimindex \\
& A\_27b)\ (c\_2Ebool\_2Ethe\_value\ A\_27b)))\ V4n))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ (ap \\
& (ap\ c\_2Earithmetic\_2E\_2B\ V2l)\ V4n))\ V0w))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2EfcP\_2Ecart \\
& 2\ A\_27a). (\forall V1h \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2EfcP\_2Edimindex\ A\_27a)\ ( \\
& c\_2Ebool\_2Ethe\_value\ A\_27a)))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ V1h)) \Rightarrow \\
& ((ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_bits\ A\_27a)\ V1h)\ c\_2Enum\_2E0) \\
& V0w) = V0w)))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0h \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (\forall V2m\_27 \in \\
& \quad ty\_2Enum\_2Enum. (\forall V3l \in ty\_2Enum\_2Enum. (\forall V4s \in ty\_2Enum\_2Enum. \\
& \quad (\forall V5w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). (((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V3l)\ V1m)) \wedge ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V2m\_27)\ V0h)) \wedge \\
& \quad ((V2m\_27 = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1m)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) \wedge (V4s = \\
& \quad (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2m\_27)\ V3l)))))) \Rightarrow ((ap\ (ap\ (c\_2Ewords\_2Eword\_or \\
& \quad A\_27b)\ (ap\ (ap\ (c\_2Ewords\_2Eword\_lsl\ A\_27b)\ (ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_extract \\
& \quad A\_27a\ A\_27b)\ V0h)\ V2m\_27)\ V5w))\ V4s))\ (ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_extract \\
& \quad A\_27a\ A\_27b)\ V1m)\ V3l)\ V5w)) = (ap\ (ap\ (ap\ (c\_2Ewords\_2Eword\_extract \\
& \quad A\_27a\ A\_27b)\ (ap\ (ap\ c\_2Earithmetic\_2EMIN\ V0h)\ (ap\ (ap\ c\_2Earithmetic\_2EMIN \\
& \quad (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap \\
& \quad (c\_2Efc\_2Edimindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_value\ A\_27b)))) \\
& \quad V3l))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ ( \\
& \quad c\_2Efc\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a)))\ ( \\
& \quad ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) \\
& \quad V3l)\ V5w)))))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& ((ap\ (c\_2Efc\_2Edimindex\ ty\_2Eone\_2Eone)\ (c\_2Ebool\_2Ethe\_value \\
& \quad ty\_2Eone\_2Eone)) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& ((ap\ (c\_2Efc\_2Edimindex\ (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit1 \\
& \quad (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))))\ (c\_2Ebool\_2Ethe\_value \\
& \quad (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone)))))) = \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& ((ap\ (c\_2Efc\_2Edimindex\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0 \\
& \quad (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0 \\
& \quad ty\_2Eone\_2Eone))))))\ (c\_2Ebool\_2Ethe\_value\ (ty\_2Efc\_2Ebit0 \\
& \quad (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0 \\
& \quad (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))))))))) = (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)))))))))
\end{aligned} \tag{91}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0x \in (ty\_2Efc\_2Ecart\ 2\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0 \\ & (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0 \\ & ty\_2Eone\_2Eone)))))).((ap\ c\_2Emachine\_iee\_2Efloat\_to\_fp64 \\ & (ap\ c\_2Emachine\_iee\_2Efp64\_to\_float\ V0x)) = V0x)) \end{aligned}$$