

thm_2Emachine_ieee_2Efp16_isNormal
(TMbG2cbx5kYt7QmWAMCUxyJxcWSZPaX5Yqp)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E$

Definition 8 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 10 We define $c_Ebit_EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_Earithmetic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 11 We define $c_Ebit_EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define c_Ebit_EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 13 We define c_Ebit_EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efc_2Efinite_image A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Efc_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 14 We define c_Ebool_2EF to be $(ap (c_Ebool_2E.21 2) (\lambda V0t \in 2.V0t))$.

Definition 15 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow Q)$ of type ι .

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 18 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge$ of type $\iota \Rightarrow \iota$).

Definition 19 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 20 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 22 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (15)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (16)$$

Definition 23 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a$

Definition 24 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 25 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcp_2EFCP$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2EUINT_MAX A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (17)$$

Definition 26 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap (c_2Ewords_2En2w A_27a) (ap (c_2Ew$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Ebinary_ieee_2Efloat A0 A1) \quad (18)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent A_27t A_27w \in ((ty_2Efcp_2Ecart 2 A_27w)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)}) \quad (19)$$

Definition 27 We define $c_Ebinary_ieee_Efloat_is_normal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebin$

Definition 28 We define $c_Ecombin_Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $ty_Eone_Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_Eone_Eone \quad (20)$$

Let $ty_Efc_Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_Efc_Ebit0\ A0) \quad (21)$$

Let $ty_Efc_Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_Efc_Ebit1\ A0) \quad (22)$$

Let $c_Ebool_EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ebool_EARB\ A_27a \in A_27a \quad (23)$$

Definition 29 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 30 We define $c_Earithmetic_EMIN$ to be $\lambda V0m \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum$

Definition 31 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2)\ (\lambda V2t \in$

Definition 32 We define $c_Earithmetic_E_3C_3D$ to be $\lambda V0m \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum$

Definition 33 We define $c_Ewords_Eword_bits$ to be $\lambda A_27a : \iota.\lambda V0h \in ty_Eenum_Eenum.\lambda V1l \in ty_Eenum_Eenum$

Definition 34 We define c_Ebit_ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_Eenum_Eenum.(ap\ (ap\ (ap\ (c_Ebool$

Let $c_Esum_num_ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_ESUM \in ((ty_Eenum_Eenum^{(ty_Eenum_Eenum^{ty_Eenum_Eenum})})^{ty_Eenum_Eenum}) \quad (24)$$

Definition 35 We define c_Ewords_Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).(ap\ (ap\ c$

Definition 36 We define c_Ewords_Ew2w to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a$

Definition 37 We define $c_Ewords_Eword_extract$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0h \in ty_Eenum_Eenum$

Definition 38 We define $c_Ecombin_EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Let $c_Ebinary_ieee_Efloat_Significand_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27v. \\ & nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_Significand_fupd \\ & A_27t\ A_27u\ A_27w \in (((ty_Ebinary_ieee_Efloat\ A_27u\ A_27w)^{(ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)})^{(ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)})^{(ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)} \end{aligned} \quad (25)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x. \\ & nonempty\ A_27x \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A_27t \\ & A_27w\ A_27x \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Efloat_Exponent_fupd\ A_27t\ A_27w)}) \\ & \end{aligned} \quad (26)$$

Let $c_2Ebinary_ieee_2Efloat_Sign_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign_fupd\ A_27t \\ & A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Efloat_Sign_fupd\ A_27t\ A_27w)}) \\ & \end{aligned} \quad (27)$$

Definition 39 We define $c_2Emachine_ieee_2Efp16_to_float$ to be $\lambda V0w \in (ty_2Efloat_Exponent_fupd\ 2\ (ty_2Efloat_Sign_fupd\ 2\ w))$

Definition 40 We define $c_2Emachine_ieee_2Efp16_isNormal$ to be $(ap\ (ap\ (c_2Efloat_Exponent_fupd\ 2\ w)))$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t \\ & A_27t\ A_27w \in ((ty_2Efloat_Exponent_fupd\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \\ & \end{aligned} \quad (28)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \\ & \end{aligned} \quad (29)$$

Definition 41 We define $c_2Ewords_2Eword_lsl$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efloat_Exponent_fupd\ 2\ A_27a).\lambda V1x \in (ty_2Efloat_Significand\ A_27a\ w)$

Definition 42 We define $c_2Ewords_2Eword_or$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efloat_Exponent_fupd\ 2\ A_27a).\lambda V1x \in (ty_2Efloat_Significand\ A_27a\ v)$

Definition 43 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b.f\ x))$

Definition 44 We define $c_2Ewords_2Eword_join$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0v \in (ty_2Efloat_Exponent_fupd\ 2\ A_27a).\lambda V1x \in (ty_2Efloat_Significand\ A_27a\ v)$

Definition 45 We define $c_2Ewords_2Eword_concat$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0v \in (ty_2Efloat_Exponent_fupd\ 2\ A_27a).\lambda V1x \in (ty_2Efloat_Significand\ A_27a\ v)$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign\ A_27t \\ & A_27t\ A_27w \in ((ty_2Efloat_Exponent_fupd\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \\ & \end{aligned} \quad (30)$$

Definition 46 We define $c_2Emachine_ieee_2Efloat_to_fp16$ to be $\lambda V0x \in (ty_2Ebinary_ieee_2Efloat\ (ty_2Efloat_Exponent_fupd\ 2\ (ty_2Efloat_Sign_fupd\ 2\ w)))$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\
& \text{nonempty } A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\
& (\forall V2x \in A_27c.((\text{ap } (\text{ap } (\text{ap } (\text{c_2Ecombin_2Eo } A_27c } A_27b } A_27a) \\
& V0f) V1g) V2x) = (\text{ap } V0f (\text{ap } V1g V2x))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (\text{ty_2Ebinary_ieee_2Efloat } (\text{ty_2Efc_2Ebit0 } (\\
& \text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone}))) (\text{ty_2Efc_2Ebit1 } \\
& (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone}))).((\text{ap } \text{c_2Emachine_ieee_2Efp16_to_float} \\
& (\text{ap } \text{c_2Emachine_ieee_2Efloat_to_fp16 } V0x)) = V0x))
\end{aligned} \tag{32}$$

Theorem 1

$$\begin{aligned}
& ((\forall V0a \in (\text{ty_2Ebinary_ieee_2Efloat } (\text{ty_2Efc_2Ebit0 } \\
& (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone}))) (\text{ty_2Efc_2Ebit1 } \\
& (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone}))).((p (\text{ap } \text{c_2Emachine_ieee_2Efp16_isNormal} \\
& (\text{ap } \text{c_2Emachine_ieee_2Efloat_to_fp16 } V0a))) \Leftrightarrow (p (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_is_normal} \\
& (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone}))) \\
& (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone})))) V0a)))) \wedge \\
& (\forall V1a \in \text{ty_2Enum_2Enum}.((p (\text{ap } \text{c_2Emachine_ieee_2Efp16_isNormal} \\
& (\text{ap } (\text{c_2Ewords_2En2w } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } \\
& (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone})))))) V1a))) \Leftrightarrow (p (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_is_normal} \\
& (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone}))) \\
& (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone})))) (\text{ap } \text{c_2Emachine_ieee_2Efp16_to_float} \\
& (\text{ap } (\text{c_2Ewords_2En2w } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } \\
& (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone})))))) V1a))))))
\end{aligned}$$