

thm_2Emachine_ieee_2Efp16_lessEqual
(TMXS7PWbY813ttxUE2FDFigpNA5moMvLrbj)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E$

Definition 8 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 10 We define $c_Ebit_EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_Earithmetic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 11 We define $c_Ebit_EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define c_Ebit_EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 13 We define c_Ebit_EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2EfcP_2Efinite_image A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2EfcP_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 14 We define c_Ebool_2EF to be $(ap (c_Ebool_2E.21 2)) (\lambda V0t \in 2.V0t)$.

Definition 15 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V0t1 V2t2)))$

Definition 18 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge P x)$) of type $\iota \Rightarrow \iota$.

Definition 19 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40) P)))$

Definition 20 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (c_2Eprim_rec) m n)$

Definition 21 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C) P))$

Definition 22 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40) (A_27a^{ty_2Enum_2Enum}))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (15)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (16)$$

Definition 23 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b)$

Definition 24 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efcp_2EFCP) g))$

Definition 25 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcp_2EFCP) n)$

Let $ty_2Ebinary_ieee_2Efloat_compare : \iota$ be given. Assume the following.

$$nonempty ty_2Ebinary_ieee_2Efloat_compare \quad (17)$$

Let $c_2Ebinary_ieee_2EUN : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EUN \in ty_2Ebinary_ieee_2Efloat_compare \quad (18)$$

Let $c_2Ebinary_ieee_2EGT : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EGT \in ty_2Ebinary_ieee_2Efloat_compare \quad (19)$$

Let $c_2Ebinary_ieee_2ELT : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2ELT \in ty_2Ebinary_ieee_2Efloat_compare \quad (20)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (21)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (22)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (23)$$

Definition 26 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Let $c_2Ebinary_ieee_2EEQ : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EEQ \in ty_2Ebinary_ieee_2Efloat_compare \quad (24)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (25)$$

Let $ty_2Ebinary_ieee_2Efloat_value : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_value \quad (26)$$

Let $c_2Ebinary_ieee_2Efloat_value_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebinary_ieee_2Efloat_value_CASE\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebinary_ieee_2Efloat_value} \quad (27)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (28)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (29)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (30)$$

Definition 27 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E40\ ($

Let $c_2Erealax_2Etreal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
 (31)

Definition 28 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$
 (32)

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal})$$
 (33)

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})$$
 (34)

Definition 29 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2EboC)))$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum})$$
 (35)

Definition 30 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap\ (ap\ (ap\ (c_2EboC)))$

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})$$
 (36)

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
 (37)

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})})$$
 (38)

Definition 31 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 32 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})$$
 (39)

Definition 33 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 34 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (40)$$

Definition 35 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (41)$$

Let $c_2Ebinary_ieeef_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieeef_2Efloat_Exponent\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_ieeef_2Efloat\ A_27t\ A_27w)}) \quad (42)$$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (43)$$

Definition 36 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal.$

Definition 37 We define $c_2Ebinary_ieeef_2Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Let $c_2Ebinary_ieeef_2Efloat : \iota$ be given. Assume the following.

$$c_2Ebinary_ieeef_2Efloat \in (ty_2Ebinary_ieeef_2Efloat_value^{ty_2Erealax_2Ereal}) \quad (44)$$

Let $c_2Ebinary_ieeef_2ENaN : \iota$ be given. Assume the following.

$$c_2Ebinary_ieeef_2ENaN \in ty_2Ebinary_ieeef_2Efloat_value \quad (45)$$

Let $c_2Ebinary_ieeef_2EInfinity : \iota$ be given. Assume the following.

$$c_2Ebinary_ieeef_2EInfinity \in ty_2Ebinary_ieeef_2Efloat_value \quad (46)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (47)$$

Definition 38 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ (c_2Ew$

Definition 39 We define $c_2Ebinary_ieee_2Efloat_value$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (48)$$

Definition 40 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (49)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (50)$$

Definition 41 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_$

Definition 42 We define $c_2Ebinary_ieee_2Efloat_compare$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Let $c_2Ebinaary_ieee_2Efloat_compare2num : \iota$ be given. Assume the following.

$$c_2Ebinaary_ieee_2Efloat_compare2num \in (ty_2Enum_2Enum^{ty_2Ebinaary_ieee_2Efloat_compare}) \quad (51)$$

Definition 43 We define $c_2Ebinary_ieee_2Efloat_compare_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_2Ebinaary_$

Definition 44 We define $c_2Ebinary_ieee_2Efloat_less_equal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebin$

Let $ty_2EfcP_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit0\ A0) \quad (52)$$

Let $ty_2EfcP_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit1\ A0) \quad (53)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (54)$$

Definition 45 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2En$

Definition 46 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 47 We define `c_2Earithmic_2E_3C_3D` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 48 We define `c_2Ewords_2Eword_bits` to be $\lambda A_27a : \iota.\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum$

Definition 49 We define `c_2Ewords_2Ew2w` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a)$

Definition 50 We define `c_2Ecombin_2Eo` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in ty_2Enum_2Enum$

Definition 51 We define `c_2Ewords_2Eword_extract` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0h \in ty_2Enum_2Enum$

Definition 52 We define `c_2Ecombin_2EK` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Let `c_2Ebinary_2IEEE_2Efloat_2Esignificand_2E_fupd` : $\iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27v.nonempty\ A_27v \Rightarrow \\ & \quad nonempty\ A_27w \Rightarrow c_2Ebinary_2IEEE_2Efloat_2Esignificand_2E_fupd\ A_27t\ A_27u\ A_27v \\ & \quad A_27t\ A_27u\ A_27w \in (((ty_2Ebinary_2IEEE_2Efloat\ A_27u\ A_27w)^{(ty_2Ebinary_2IEEE_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_2IEEE_2Efloat\ A_27t\ A_27u)}) \end{aligned} \quad (55)$$

Let `c_2Ebinary_2IEEE_2Efloat_2Eexponent_2E_fupd` : $\iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x.nonempty\ A_27x \Rightarrow \\ & \quad nonempty\ A_27y \Rightarrow c_2Ebinary_2IEEE_2Efloat_2Eexponent_2E_fupd\ A_27t\ A_27w\ A_27x \\ & \quad A_27w\ A_27x \in (((ty_2Ebinary_2IEEE_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_2IEEE_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_2IEEE_2Efloat\ A_27t\ A_27x)}) \end{aligned} \quad (56)$$

Let `c_2Ebinary_2IEEE_2Efloat_2Esign_2E_fupd` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_2IEEE_2Efloat_2Esign_2E_fupd\ A_27t\ A_27w \\ & \quad A_27t\ A_27w \in (((ty_2Ebinary_2IEEE_2Efloat\ A_27t\ A_27w)^{(ty_2Ebinary_2IEEE_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_2IEEE_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (57)$$

Definition 53 We define `c_2Emachine_2IEEE_2Efp16_2E_to_2Efloat` to be $\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ (ty_2EfcP_2Ecart\ 2\ (ty_2EfcP_2Ecart\ 2\ A_27a)))$

Definition 54 We define `c_2Emachine_2IEEE_2Efp16_2E_lessEqual` to be $\lambda V0a \in (ty_2EfcP_2Ecart\ 2\ (ty_2EfcP_2Ecart\ 2\ (ty_2EfcP_2Ecart\ 2\ A_27a)))$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \end{aligned} \quad (58)$$

Definition 55 We define `c_2Ewords_2Eword_lsl` to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1u \in ty_2Enum_2Enum$

Definition 56 We define `c_2Ewords_2Eword_or` to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1u \in ty_2Enum_2Enum$

Definition 57 We define `c_2Ebool_2ELET` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b.V0x))$

Definition 58 We define `c2Ewords_2Eword_join` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2)$

Definition 59 We define `c2Ewords_2Eword_concat` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2)$

Definition 60 We define `c2Emachine_ieee_2Efloat_to_fp16` to be $\lambda V0x \in (ty_2Ebinary_ieee_2Efloat\ t)$

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty_2Ebinary_ieee_2Efloat\ (ty_2Efc_2Ebit0\ (\\ & \quad ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0\ ty_2Eone_2Eone)))\ (ty_2Efc_2Ebit1 \\ & \quad ty_2Efc_2Ebit0\ ty_2Eone_2Eone)))) \cdot ((ap\ c2Emachine_ieee_2Efp16_to_float \\ & \quad (ap\ c2Emachine_ieee_2Efloat_to_fp16\ V0x)) = V0x) \end{aligned} \tag{59}$$

Theorem 1

$$\begin{aligned}
& ((\forall V0b \in (ty_2Ebinary_ieee_2Efloat (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) (ty_2Efc_2Ebit1 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone))).(\forall V1a \in (ty_2Ebinary_ieee_2Efloat \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))).((p (ap \\
& (ap c_2Emachine_ieee_2Efp16_lessEqual (ap c_2Emachine_ieee_2Efloat_to_fp16 \\
& V1a)) (ap c_2Emachine_ieee_2Efloat_to_fp16 V0b)))) \Leftrightarrow (p (ap \\
& (ap (c_2Ebinary_ieee_2Efloat_less_equal (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) (ty_2Efc_2Ebit1 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) V1a) V0b)))) \wedge ((\forall V2b \in \\
& ty_2Enum_2Enum.(\forall V3a \in (ty_2Ebinary_ieee_2Efloat (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) (ty_2Efc_2Ebit1 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone))).((p (ap (ap c_2Emachine_ieee_2Efp16_lessEqual \\
& (ap c_2Emachine_ieee_2Efloat_to_fp16 V3a)) (ap (c_2Ewords_2En2w \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& ty_2Eone_2Eone)))))) V2b)))) \Leftrightarrow (p (ap (ap (c_2Ebinary_ieee_2Efloat_less_equal \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) V3a) (ap \\
& c_2Emachine_ieee_2Efp16_to_float (ap (c_2Ewords_2En2w (\\
& ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& ty_2Eone_2Eone)))))) V2b)))))) \wedge ((\forall V4b \in (ty_2Ebinary_ieee_2Efloat \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))).(\forall V5a \in \\
& ty_2Enum_2Enum.((p (ap (ap c_2Emachine_ieee_2Efp16_lessEqual \\
& (ap (c_2Ewords_2En2w (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) V5a)) (ap c_2Emachine_ieee_2Efloat_to_fp16 \\
& V4b)))) \Leftrightarrow (p (ap (ap (c_2Ebinary_ieee_2Efloat_less_equal (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) (ty_2Efc_2Ebit1 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) (ap c_2Emachine_ieee_2Efp16_to_float \\
& (ap (c_2Ewords_2En2w (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) V5a))) V4b)))))) \wedge ((\forall V6b \in \\
& ty_2Enum_2Enum.(\forall V7a \in ty_2Enum_2Enum.((p (ap (ap c_2Emachine_ieee_2Efp16_lessEqual \\
& (ap (c_2Ewords_2En2w (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) V7a)) (ap (c_2Ewords_2En2w \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& ty_2Eone_2Eone)))))) V6b)))) \Leftrightarrow (p (ap (ap (c_2Ebinary_ieee_2Efloat_less_equal \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))) (ap c_2Emachine_ieee_2Efp16_to_float \\
& (ap (c_2Ewords_2En2w (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) V7a))) (ap c_2Emachine_ieee_2Efp16_to_float \\
& (ap (c_2Ewords_2En2w (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) V6b)))))))))
\end{aligned}$$