

# thm\_2Emachine\_\_ieee\_2Efp16\_\_lessThan (TMR- wkKbZX5wXDyrksjvLxhNVGpgt36VdBJW)

October 26, 2020

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let  $c\_2Earithmetic\_2E2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2E$

**Definition 8** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2E$

Let  $c\_Earithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 10** We define  $c\_Ebit\_EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_Earithmetic\_EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 11** We define  $c\_Ebit\_EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebit\_EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 13** We define  $c\_Ebit\_EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Let  $ty\_2EfcP\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2EfcP\_2Efinite\_image A0) \quad (11)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (12)$$

Let  $c\_Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Ebool\_2Ethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (13)$$

Let  $c\_2EfcP\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2EfcP\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (14)$$

**Definition 14** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E.21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 16** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

**Definition 17** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.(ap (c_2Emin_2E_3D_3D_3E V2t) c_2Ebool_2E_7E))))$

**Definition 18** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A.\lambda y.p (ap P y)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 19** We define `c_2Ebool_2E_3F` to be  $\lambda A_{.27a} : \iota.(\lambda V0P \in (2^{A_{.27a}}).(ap V0P (ap (c_2Emin_2E_40 A_{.27a}) P)))$

**Definition 20** We define `c_2Eprim_rec_2E_3C` to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c_2Emin_2E_40 V0m) V1n)$

**Definition 21** We define `c_2Ebool_2E_3F_21` to be  $\lambda A_{.27a} : \iota.(\lambda V0P \in (2^{A_{.27a}}).(ap (ap c_2Ebool_2E_2F_5C A_{.27a}) P)))$

**Definition 22** We define `c_2Efcp_2Efinite_index` to be  $\lambda A_{.27a} : \iota.(ap (c_2Emin_2E_40 A_{.27a})^{ty\_2Enum\_2Enum})$

Let `ty_2Efcp_2Ecart` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart A0 A1) \quad (15)$$

Let `c_2Efcp_2Edest_cart` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow c\_2Efcp\_2Edest\_cart A_{.27a} A_{.27b} \in ((A_{.27a}^{(ty\_2Efcp\_2Efinite\_image A_{.27b})})^{(ty\_2Efcp\_2Ecart A_{.27a} A_{.27b})}) \quad (16)$$

**Definition 23** We define `c_2Efcp_2Efcp_index` to be  $\lambda A_{.27a} : \iota.\lambda A_{.27b} : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart A_{.27a} A_{.27b})$

**Definition 24** We define `c_2Efcp_2EFCP` to be  $\lambda A_{.27a} : \iota.\lambda A_{.27b} : \iota.(\lambda V0g \in (A_{.27a}^{ty\_2Enum\_2Enum}))^{(ap (c_2Efcp_2Efcp_index A_{.27a} A_{.27b}) g)}$

**Definition 25** We define `c_2Ewords_2En2w` to be  $\lambda A_{.27a} : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c_2Efcp_2EFCP A_{.27a} A_{.27a}) V0n)$

Let `ty_2Ebinary_ieee_2Efloat_compare` :  $\iota$  be given. Assume the following.

$$nonempty ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (17)$$

Let `c_2Ebinary_ieee_2ELT` :  $\iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ELT \in ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (18)$$

Let `c_2Ebinary_ieee_2EUN` :  $\iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EUN \in ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (19)$$

Let `c_2Ebinary_ieee_2EGT` :  $\iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EGT \in ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (20)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (21)$$

Let  $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_ieee\_2Efloat\ A0\ A1) \quad (22)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efc\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (23)$$

**Definition 26** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

Let  $c\_2Ebinary\_ieee\_2EEQ : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EEQ \in ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (24)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (25)$$

Let  $ty\_2Ebinary\_ieee\_2Efloat\_value : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_value \quad (26)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_value\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_value\_CASE\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_ieee\_2Efloat\_value} \quad (27)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (28)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (29)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (30)$$

**Definition 27** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Emin\_2E40\ ($

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})$$
 (31)

**Definition 28** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})$$
 (32)

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal})$$
 (33)

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})$$
 (34)

**Definition 29** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2EboC)))$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum})$$
 (35)

**Definition 30** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).(ap\ (ap\ (ap\ (c\_2EboC)))$

Let  $c\_2Erealax\_2Etreal\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})$$
 (36)

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})$$
 (37)

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})})$$
 (38)

**Definition 31** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 32** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS)$

Let  $c\_2Erealax\_2Etreal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})$$
 (39)

**Definition 33** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 34** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (40)$$

**Definition 35** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Ewords\_2EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (41)$$

Let  $c\_2Ebinary\_ieec\_2Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieec\_2Efloat\_Exponent\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_ieec\_2Efloat\ A\_27t\ A\_27w)}) \quad (42)$$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (43)$$

**Definition 36** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal.$

**Definition 37** We define  $c\_2Ebinary\_ieec\_2Efloat\_to\_real$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

Let  $c\_2Ebinary\_ieec\_2Efloat : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieec\_2Efloat \in (ty\_2Ebinary\_ieec\_2Efloat\_value^{ty\_2Erealax\_2Ereal}) \quad (44)$$

Let  $c\_2Ebinary\_ieec\_2ENaN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieec\_2ENaN \in ty\_2Ebinary\_ieec\_2Efloat\_value \quad (45)$$

Let  $c\_2Ebinary\_ieec\_2EInfinity : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieec\_2EInfinity \in ty\_2Ebinary\_ieec\_2Efloat\_value \quad (46)$$

Let  $c\_2Ewords\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (47)$$

**Definition 38** We define  $c\_2Ewords\_2Eword\_T$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Ewords\_2En2w\ A\_27a)\ (ap\ (c\_2Ew$

**Definition 39** We define  $c\_2Ebinary\_ieee\_2Efloat\_value$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (48)$$

**Definition 40** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (49)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (50)$$

**Definition 41** We define  $c\_2Epair\_2Epair\_CASE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair$

**Definition 42** We define  $c\_2Ebinary\_ieee\_2Efloat\_compare$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

**Definition 43** We define  $c\_2Ebinary\_ieee\_2Efloat\_less\_than$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

Let  $ty\_2EfcP\_2Ebit0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2EfcP\_2Ebit0\ A0) \quad (51)$$

Let  $ty\_2EfcP\_2Ebit1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2EfcP\_2Ebit1\ A0) \quad (52)$$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (53)$$

**Definition 44** We define  $c\_2Earithmetic\_2EMIN$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 45** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 46** We define  $c\_2Earithmetic\_2E\_3C\_2D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 47** We define  $c\_2Ewords\_2Eword\_bits$  to be  $\lambda A\_27a : \iota.\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2$

**Definition 48** We define  $c\_2Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a$

**Definition 49** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$



Assume the following.

$$\begin{aligned}
 & (\forall V0x \in (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit0 ( \\
 & \quad ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc\_2Ebit1 \\
 & (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))).((ap\ c\_2Emachine\_ieee\_2Efp16\_to\_float \\
 & \quad (ap\ c\_2Emachine\_ieee\_2Efloat\_to\_fp16\ V0x)) = V0x))
 \end{aligned}
 \tag{58}$$

**Theorem 1**

$$\begin{aligned}
& ((\forall V0b \in (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc2Ebit0 \\
& (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc2Ebit1 \\
& (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))).(\forall V1a \in (ty\_2Ebinary\_ieee\_2Efloat \\
& (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) \\
& (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))).((p (ap \\
& (ap c\_2Emachine\_ieee\_2Efp16\_lessThan (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 \\
& V1a)) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 V0b)))) \Leftrightarrow (p (ap \\
& (ap (c\_2Ebinary\_ieee\_2Efloat\_less\_than (ty\_2Efc2Ebit0 \\
& (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc2Ebit1 \\
& (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) V1a) V0b)))))) \wedge ((\forall V2b \in \\
& ty\_2Enum\_2Enum.(\forall V3a \in (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc2Ebit0 \\
& (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc2Ebit1 \\
& (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))).((p (ap (ap c\_2Emachine\_ieee\_2Efp16\_lessThan \\
& (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 V3a)) (ap (c\_2Ewords\_2En2w \\
& (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 \\
& ty\_2Eone\_2Eone)))))) V2b)))) \Leftrightarrow (p (ap (ap (c\_2Ebinary\_ieee\_2Efloat\_less\_than \\
& (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) \\
& (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) V3a) (ap \\
& c\_2Emachine\_ieee\_2Efp16\_to\_float (ap (c\_2Ewords\_2En2w ( \\
& ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 \\
& ty\_2Eone\_2Eone)))))) V2b)))))) \wedge ((\forall V4b \in (ty\_2Ebinary\_ieee\_2Efloat \\
& (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) \\
& (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))).(\forall V5a \in \\
& ty\_2Enum\_2Enum.((p (ap (ap c\_2Emachine\_ieee\_2Efp16\_lessThan \\
& (ap (c\_2Ewords\_2En2w (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 \\
& (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone)))))) V5a)) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 \\
& V4b)))) \Leftrightarrow (p (ap (ap (c\_2Ebinary\_ieee\_2Efloat\_less\_than (ty\_2Efc2Ebit0 \\
& (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc2Ebit1 \\
& (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) (ap c\_2Emachine\_ieee\_2Efp16\_to\_float \\
& (ap (c\_2Ewords\_2En2w (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 \\
& (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone)))))) V5a)) V4b)))))) \wedge ((\forall V6b \in \\
& ty\_2Enum\_2Enum.(\forall V7a \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Emachine\_ieee\_2Efp16\_lessThan \\
& (ap (c\_2Ewords\_2En2w (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 \\
& (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone)))))) V7a)) (ap (c\_2Ewords\_2En2w \\
& (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 \\
& ty\_2Eone\_2Eone)))))) V6b)))) \Leftrightarrow (p (ap (ap (c\_2Ebinary\_ieee\_2Efloat\_less\_than \\
& (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) \\
& (ty\_2Efc2Ebit1 (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone))) (ap c\_2Emachine\_ieee\_2Efp16\_to\_float \\
& (ap (c\_2Ewords\_2En2w (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 \\
& (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone)))))) V7a)) (ap c\_2Emachine\_ieee\_2Efp16\_to\_float \\
& (ap (c\_2Ewords\_2En2w (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 (ty\_2Efc2Ebit0 \\
& (ty\_2Efc2Ebit0 ty\_2Eone\_2Eone)))))) V6b)))))))))
\end{aligned}$$