

thm\_2Emachine\_ieee\_2Efp16\_mul\_sub\_with\_flags  
(TM-  
RKGu42oAgEvZfmPhPigfg4cHSgmXLSNWC)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Enum\_2ESUC\_REP\ m))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 10** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 11** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 13** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Let  $ty\_2Efc\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efc\_2Efinite\_image A0) \quad (11)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (12)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (13)$$

Let  $c\_2Efc\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Efc\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (14)$$

**Definition 14** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 16** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$ .

**Definition 17** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$ .

**Definition 18** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A.\lambda y.y \in A))$  of type  $\iota \Rightarrow \iota$ .

**Definition 19** We define `c_2Ebool_2E_3F` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P (ap (c_2Emin_2E_40\ A_27a))))$ .

**Definition 20** We define `c_2Eprim_rec_2E_3C` to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$ .

**Definition 21** We define `c_2Ebool_2E_3F_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap\ c_2Ebool_2E_2F_5C\ V0P)))$ .

**Definition 22** We define `c_2Efcp_2Efinite_index` to be  $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty\_2Enum\_2Enum})))$ .

Let `ty_2Efcp_2Ecart` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart\ A0\ A1) \quad (15)$$

Let `c_2Efcp_2Edest_cart` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c\_2Efcp\_2Edest\_cart\ A_27a\ A_27b \in ((A_27a^{(ty\_2Efcp\_2Efinite\_image\ A_27b)})^{(ty\_2Efcp\_2Ecart\ A_27a\ A_27b)}) \quad (16)$$

**Definition 23** We define `c_2Efcp_2Efcp_index` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A_27a\ A_27b)$ .

**Definition 24** We define `c_2Efcp_2EFCP` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty\_2Enum\_2Enum}).(ap (c_2Efcp_2Efcp\_index\ A_27a\ A_27b\ g)))$ .

**Definition 25** We define `c_2Ewords_2En2w` to be  $\lambda A_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c_2Efcp_2EFCP\ A_27a\ n))$ .

Let `ty_2Eone_2Eone` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (17)$$

Let `ty_2Ebinary_ieee_2Efloat` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efloat\ A0\ A1) \quad (18)$$

Let `c_2Ebinary_ieee_2Efloat_Sign` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign\ A_27t\ A_27w \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat\ A_27t\ A_27w)}) \quad (19)$$

**Definition 26** We define  $c\_2Ewords\_2Eword\_xor$  to be  $\lambda A\_27a : \iota. \lambda V0v \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a). \lambda V1$   
Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (20)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (21)$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (22)$$

Let  $c\_2Ebinaary\_2ieee\_2Efloat\_2Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t. nonempty\ A\_27t \Rightarrow \forall A\_27w. nonempty\ A\_27w \Rightarrow c\_2Ebinaary\_2ieee\_2Efloat\_2Significand\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinaary\_2ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (23)$$

**Definition 27** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 28** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebooc$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (24)$$

**Definition 29** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a). (ap\ (ap\ c$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (25)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (26)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (27)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Emin\_2E40\ (t$

Let  $c\_2Erealax\_2Etreal\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (28)$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (29)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (30)$$

**Definition 31** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 32** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etreal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (31)$$

**Definition 33** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 34** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etreal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (32)$$

**Definition 35** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Ewords\_2EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_MAX\ A\_27a \in (ty\_2Enum\_2Enum)^{(ty\_2Ebool\_2Eitself\ A\_27a)} \quad (33)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (34)$$

Let  $c\_2Erealax\_2Etreal\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (35)$$

**Definition 36** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

**Definition 37** We define  $c\_2Ebinary\_ieee\_2Efloat\_to\_real$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let  $ty\_2Ebinary\_ieee\_2Efloat\_value : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_value \quad (36)$$

Let  $c\_2Ebinary\_ieee\_2Efloat : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat \in (ty\_2Ebinary\_ieee\_2Efloat\_value^{ty\_2Erealax\_2Ereal}) \quad (37)$$

Let  $c\_2Ebinary\_ieee\_2ENaN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ENaN \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (38)$$

Let  $c\_2Ebinary\_ieee\_2EInfinity : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EInfinity \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (39)$$

Let  $c\_2Ewords\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (40)$$

**Definition 38** We define  $c\_2Ewords\_2Eword\_T$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Ewords\_2En2w\ A\_27a)\ (ap\ (c\_2E$

**Definition 39** We define  $c\_2Ebinary\_ieee\_2Efloat\_value$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let  $c\_2Ebinary\_ieee\_2Efloat\_value\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_value\_CASE\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_ieee\_2Efloat\_value} \quad (41)$$

**Definition 40** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_infinite$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

**Definition 41** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in$

**Definition 42** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let  $ty\_2Ebinary\_ieee\_2ERounding : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2ERounding \quad (42)$$

Let  $c\_2Ebinary\_ieee\_2ERoundTowardNegative : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ERoundTowardNegative \in ty\_2Ebinary\_ieee\_2ERounding \quad (43)$$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh$$
 (44)

**Definition 43** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 44** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 45** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_2Ebool\_2ECONJ$

Let  $c\_2Ebinary\_ieee\_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Elargest \\ & A\_27t A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself (ty\_2Epair\_2Eprod A\_27t A\_27w))}) \end{aligned} \quad (45)$$

**Definition 46** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_finite$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ & A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (46)$$

**Definition 47** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ & A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \end{aligned} \quad (47)$$

**Definition 48** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

**Definition 49** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

**Definition 50** We define  $c\_2Ebinary\_ieee\_2Eis\_closest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{(ty\_2Ebina$

**Definition 51** We define  $c\_2Ebinary\_ieee\_2Eclosest\_such$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Ebina$

**Definition 52** We define  $c\_2Ebinary\_ieee\_2Eclosest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap (c\_2Ebina$

Let  $c\_2Ebinary\_ieee\_2Efloat\_top : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebina$$

**Definition 53** We define  $c\_2Ereal\_2Ereal\_gt$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ebinary\_ieee\_2Efloat\_bottom : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebina$$

**Definition 54** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27w$

Let  $c\_2Ebinary\_ieee\_2Efloat\_minus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_minus\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (50)$$

**Definition 55** We define  $c\_2Ereal\_2Ereal\_ge$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ebinary\_ieee\_2Efloat\_plus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_plus\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (51)$$

Let  $c\_2Ebinary\_ieee\_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Ethreshold\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (52)$$

**Definition 56** We define  $c\_2Ewords\_2Eword\_lsb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcpcart\ 2\ A\_27a).(\lambda V1$

Let  $c\_2Ebinary\_ieee\_2Erounding2num : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Erounding2num \in (ty\_2Enum\_2Enum^{ty\_2Ebinary\_ieee\_2Erounding}) \quad (53)$$

**Definition 57** We define  $c\_2Ebinary\_ieee\_2Erounding\_CASE$  to be  $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Ebinary\_ieee\_2Erounding$

**Definition 58** We define  $c\_2Ebinary\_ieee\_2Eround$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinary\_ieee\_2Eround$

Let  $c\_2Ebinary\_ieee\_2Efloat\_plus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_plus\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (54)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_minus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_minus\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (55)$$

**Definition 59** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_zero$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\_is\_zero$

**Definition 60** We define  $c\_2Ebinary\_ieee\_2Efloat\_round$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinary\_ieee\_2Efloat\_round$



Let  $ty\_2Ebinary\_ieee\_2Eflags : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Eflags \quad (56)$$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (57)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (58)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (59)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (60)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (61)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (62)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (63)$$

**Definition 61** We define  $c\_2Ebinary\_ieee\_2Eclear\_flags$  to be  $(ap\ (ap\ c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd))$

Let  $c\_2Ewords\_2EINT\_MIN : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_MIN\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (64)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (65)$$

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (66)$$

**Definition 62** We define  $c\_2Ewords\_2Eword\_2comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).$

**Definition 63** We define  $c\_2Ewords\_2Eword\_2msb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).$

**Definition 64** We define  $c\_2Ewords\_2Enzcv$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1b \in$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (67)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (68)$$

**Definition 65** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})$

**Definition 66** We define  $c\_2Ewords\_2Eword\_2ls$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1b$

**Definition 67** We define  $c\_2Ebinary\_2ieee\_2Efloat\_2round\_2with\_2flags$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode$

Let  $ty\_2Ebinary\_2ieee\_2Efp\_2op : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_2ieee\_2Efp\_2op\ A0\ A1) \quad (69)$$

Let  $c\_2Ebinary\_2ieee\_2EFP\_2MulAdd : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$A\_27t\ A\_27w \in (((((ty\_2Ebinary\_2ieee\_2Efp\_2op\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_2ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Efloat\ A\_27t\ A\_27w)}) \quad (70)$$

**Definition 68** We define  $c\_2Ebinary\_2ieee\_2Efloat\_2is\_2nan$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_2ieee\_2Efloat\_2is\_2nan\ A\_27t\ A\_27w)$

**Definition 69** We define  $c\_2Ebinary\_2ieee\_2Efloat\_2is\_2signalling$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_2ieee\_2Efloat\_2is\_2signalling\ A\_27t\ A\_27w)$

**Definition 70** We define  $c\_2Ebinary\_2ieee\_2Efloat\_2some\_2qnan$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0fp\_2op \in (ty\_2Ebinary\_2ieee\_2Efloat\_2some\_2qnan\ A\_27t\ A\_27w\ fp\_2op)$

**Definition 71** We define  $c\_2Ebinary\_2ieee\_2Einvalidop\_2flags$  to be  $(ap\ (ap\ c\_2Ebinary\_2ieee\_2Eflags\_2Invalidop\_2flags\ A\_27t\ A\_27w)\ fp\_2op)$

Let  $c\_2Ebinary\_ieee\_2EFP\_MulSub : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t A\_27w \in (((((ty\_2Ebinary\_ieee\_2Efp\_op A\_27t A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)})^{(ty\_2E})))))) \quad (71)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (72)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (73)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (74)$$

Let  $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EEXISTS A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})^{(2^{A\_27a})}) \quad (75)$$

**Definition 72** We define  $c\_2Ebinary\_ieee\_2Echeck\_for\_signalling$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0l \in (ty\_2E$

**Definition 73** We define  $c\_2Ebinary\_ieee\_2Efloat\_mul\_sub$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0mode \in ty\_2E$

**Definition 74** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 75** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A$

**Definition 76** We define  $c\_2Epair\_2E\_23\_23$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda A\_27d : \iota. \lambda V0f \in (A\_27$

Let  $ty\_2Efc\_2Ebit0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efc\_2Ebit0 A0) \quad (76)$$

Let  $ty\_2Efc\_2Ebit1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efc\_2Ebit1 A0) \quad (77)$$

**Definition 77** We define  $c\_2Earithmetic\_2EMIN$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 78** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 79** We define  $c\_2Ewords\_2Eword\_bits$  to be  $\lambda A\_27a : \iota. \lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2$

**Definition 80** We define  $c\_2Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w \in (ty\_2Efc\_2Ecart 2 A\_27a$

**Definition 81** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$ .

**Definition 82** We define  $c\_2Ewords\_2Eword\_extract$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0h \in ty\_2Enum\_2Enum$ .

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27v. \\ & nonempty\ A\_27v \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd\ A\_27t\ A\_27u\ A\_27v \in \\ & (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27u\ A\_27v)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27v)})) \end{aligned} \quad (78)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow \forall A\_27x. \\ & nonempty\ A\_27x \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd\ A\_27t\ A\_27w\ A\_27x \in \\ & (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})) \end{aligned} \quad (79)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign\_fupd : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee. \\ & A\_27t\ A\_27w \in (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})) \end{aligned} \quad (80)$$

**Definition 83** We define  $c\_2Emachine\_ieee\_2Efp16\_to\_float$  to be  $\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ (ty\_2Efc\_2Ecart\ 2\ A\_27a)).\lambda V1$ .

**Definition 84** We define  $c\_2Ewords\_2Eword\_lsl$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1$ .

**Definition 85** We define  $c\_2Ewords\_2Eword\_or$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1$ .

**Definition 86** We define  $c\_2Ewords\_2Eword\_join$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1$ .

**Definition 87** We define  $c\_2Ewords\_2Eword\_concat$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1$ .

**Definition 88** We define  $c\_2Emachine\_ieee\_2Efloat\_to\_fp16$  to be  $\lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ (ty\_2Efc\_2Ecart\ 2\ A\_27a)).\lambda V1$ .

**Definition 89** We define  $c\_2Emachine\_ieee\_2Efp16\_mul\_sub\_with\_flags$  to be  $\lambda V0mode \in ty\_2Ebinary\_ieee\_2Erounding.\lambda V1a \in (ty\_2Efc\_2Ecart\ 2\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))).\lambda V1$ .

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ (ty\_2Efc\_2Ebit0\ ( \\ & ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))))\ (ty\_2Efc\_2Ebit1 \\ & (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))).((ap\ c\_2Emachine\_ieee\_2Efp16\_to\_float \\ & (ap\ c\_2Emachine\_ieee\_2Efloat\_to\_fp16\ V0x)) = V0x) \end{aligned} \quad (81)$$



**Theorem 1**

$$\begin{aligned}
& ((\forall V0mode \in ty\_2Ebinary\_ieee\_2Erounding. (\forall V1c \in \\
& (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 \\
& (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 \\
& ty\_2Eone\_2Eone))). (\forall V2b \in (ty\_2Ebinary\_ieee\_2Efloat \\
& (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) \\
& (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))). (\forall V3a \in \\
& (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 \\
& (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 \\
& ty\_2Eone\_2Eone))). ((ap (ap (ap (ap c\_2Emachine\_ieee\_2Efp16\_mul\_sub\_with\_flags \\
V0mode) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 V3a)) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 \\
V2b)) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 V1c)) = (ap (ap \\
(ap (c\_2Epair\_2E\_23\_23 ty\_2Ebinary\_ieee\_2Eflags (ty\_2Ebinary\_ieee\_2Efloat \\
(ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) \\
(ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) ty\_2Ebinary\_ieee\_2Eflags \\
(ty\_2Efc\_2Ecart 2 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 \\
(ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))))) (c\_2Ecombin\_2EI ty\_2Ebinary\_ieee\_2Eflags)) \\
c\_2Emachine\_ieee\_2Efloat\_to\_fp16) (ap (ap (ap (ap (c\_2Ebinary\_ieee\_2Efloat\_mul\_sub \\
(ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) \\
(ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) V0mode) \\
V3a) V2b) V1c)))))) \wedge ((\forall V4mode \in ty\_2Ebinary\_ieee\_2Erounding. \\
(\forall V5c \in ty\_2Enum\_2Enum. (\forall V6b \in (ty\_2Ebinary\_ieee\_2Efloat \\
(ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) \\
(ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))). (\forall V7a \in \\
(ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 \\
(ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 \\
ty\_2Eone\_2Eone))). ((ap (ap (ap (ap c\_2Emachine\_ieee\_2Efp16\_mul\_sub\_with\_flags \\
V4mode) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 V7a)) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 \\
V6b)) (ap (c\_2Ewords\_2En2w (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 \\
(ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))))) V5c)) = \\
(ap (ap (ap (c\_2Epair\_2E\_23\_23 ty\_2Ebinary\_ieee\_2Eflags (ty\_2Ebinary\_ieee\_2Efloat \\
(ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) \\
(ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) ty\_2Ebinary\_ieee\_2Eflags \\
(ty\_2Efc\_2Ecart 2 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 \\
(ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))))) (c\_2Ecombin\_2EI ty\_2Ebinary\_ieee\_2Eflags)) \\
c\_2Emachine\_ieee\_2Efloat\_to\_fp16) (ap (ap (ap (ap (c\_2Ebinary\_ieee\_2Efloat\_mul\_sub \\
(ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) \\
(ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) V4mode) \\
V7a) V6b) (ap c\_2Emachine\_ieee\_2Efp16\_to\_float (ap (c\_2Ewords\_2En2w \\
(ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 \\
ty\_2Eone\_2Eone)))))) V5c)))))) \wedge ((\forall V8mode \in ty\_2Ebinary\_ieee\_2Erounding. \\
(\forall V9c \in (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit0 ( \\
ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc\_2Ebit1 \\
(ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))). (\forall V10b \in ty\_2Enum\_2Enum. \\
(\forall V11a \in (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit0 \\
(ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc\_2Ebit1 \\
(ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))). ((ap (ap (ap (ap c\_2Emachine\_ieee\_2Efp16\_mul\_sub\_with\_flags \\
V8mode) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 V11a)) (ap ( \\
c\_2Ewords\_2En2w (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 \\
(ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))))) V10b)) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp16 \\
V9c)) = (ap (ap (ap (c\_2Epair\_2E\_23\_23 ty\_2Ebinary\_ieee\_2Eflags \\
(ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 \\
(ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 \\
ty\_2Eone\_2Eone))) ty\_2Ebinary\_ieee\_2Eflags (ty\_2Efc\_2Ecart \\
2 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 \\
ty\_2Eone\_2Eone)))))) (c\_2Ecombin\_2EI ty\_2Ebinary\_ieee\_2Eflags)) \\
c\_2Emachine\_ieee\_2Efloat\_to\_fp16) (ap (ap (ap (ap (c\_2Ebinary\_ieee\_2Efloat\_mul\_sub \\
(ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) \\
(ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))) V8mode) \\
V11a) (ap c\_2Emachine\_ieee\_2Efp16\_to\_float (ap (c\_2Ewords\_2En2w
\end{aligned}$$