thm_2Emachine__ieee_2Efp16__to__int (TMdSzpMj1WKo2rZhdYtEUC1nQEtdH94WpDC)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (3)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (5)$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 9 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 10 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Definition 11 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 12 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 13 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{11}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{12}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{13}$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{14}$$

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 16 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 17 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 18 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 19 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 20 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 21 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 22 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap$ (c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \quad (15)$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \quad (16)$$

Definition 23 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Definition 24 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 25 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (17)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (18)$$

Definition 26 *We define* c_2Eone_2Eone *to be* $(ap$ (c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (19)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \quad (20)$$

Definition 27 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{21}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{22}$$

Definition 28 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{23}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{24}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{25}$$

Definition 29 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (t$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ t} \tag{26}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{27}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \tag{28}$$

Definition 30 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 31 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteg$

Definition 32 *We define* c_2Einteger_2ENum *to be* $\lambda V0i \in ty\_2Einteger\_2Eint.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2E$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{29}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{30}$$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{31}$$

Definition 33 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{32}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{33}$$

Definition 34 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{34}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{35}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{36}$$

Definition 35 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 36 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal_

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_} \tag{37}$$

Definition 37 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_$

Definition 38 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 39 *We define* c_2Eintreal_2Ereal__of__int *to be* $\lambda V0i \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ ($c_2Ebo

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{38}$$

Definition 40 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 41 *We define* c_2Einteger_2ELEAST__INT *to be* $\lambda V0P \in (2^{ty\_2Einteger\_2Eint}).(ap$ (c_2Emin_2E_4

Definition 42 *We define* c_2Eintreal_2EINT__FLOOR *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Einteger_2E

Definition 43 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 44 *We define* c_2Eintreal_2EINT__CEILING *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Einteger_2

Let $ty\_2Ebinary\_\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A0\ A1) \tag{39}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{40}$$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{41}$$

Definition 45 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 46 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Definition 47 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECON

Definition 48 *We define* c_2Einteger_2EABS *to be* $\lambda V0n \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Ebool_2EC

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{42}$$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{43}$$

Definition 49 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (44)$$

Definition 50 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 51 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Definition 52 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 53 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Let $ty\_2Ebinary\_\_ieee\_2Erounding : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Erounding \quad (45)$$

Let $c\_2Ebinary\_\_ieee\_2Erounding2num : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Erounding2num \in (ty\_2Enum\_2Enum^{ty\_2Ebinary\_\_ieee\_2Erounding}) \quad (46)$$

Definition 54 *We define* c_2Ebinary__ieee_2Erounding__CASE *to be* $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Ebinary\_\_ieee\_2$

Definition 55 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 56 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (47)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (48)$$

Definition 57 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Eboo

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (49)$$

Definition 58 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let $c\_2Ewords\_2EINT\_\_MAX : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ewords\_2EINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_2} \tag{50}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{51}$$

Definition 59 *We define* c_2Ebinary__ieee_2Efloat__to__real *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Let $ty\_2Ebinary\_\_ieee\_2Efloat\_\_value : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{52}$$

Let $c\_2Ebinary\_\_ieee\_2EFloat : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EFloat \in (ty\_2Ebinary\_\_ieee\_2Efloat\_\_value^{ty\_2Erealax\_2Ereal}) \tag{53}$$

Let $c\_2Ebinary\_\_ieee\_2ENaN : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2ENaN \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{54}$$

Let $c\_2Ebinary\_\_ieee\_2EInfinity : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EInfinity \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{55}$$

Let $c\_2Ewords\_2EUINT\_\_MAX : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ewords\_2EUINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{56}$$

Definition 60 *We define* c_2Ewords_2Eword__T *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ (c\_2Ew$

Definition 61 *We define* c_2Ebinary__ieee_2Efloat__value *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE\ A\_27a \in ((((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_\_ieee\_2Efloat\_\_value}) \tag{57}$$

Definition 62 *We define* c_2Ebinary__ieee_2Efloat__to__int *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebin$

Definition 63 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let $ty\_2Efcp\_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Ebit0\ A0) \quad (58)$$

Let $ty\_2Efcp\_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Ebit1\ A0) \quad (59)$$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (60)$$

Definition 64 *We define* c_2Earithmetic_2EMIN *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 65 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 66 *We define* c_2Ewords_2Eword__bits *to be* $\lambda A\_27a : \iota.\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2$

Definition 67 *We define* c_2Ewords_2Ew2w *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a$

Definition 68 *We define* c_2Ewords_2Eword__extract *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0h \in ty\_2Enum\_2Enum$

Definition 69 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27w.$$
$$nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_f$$
$$A\_27t\ A\_27u\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27u\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(}$$
$$(61)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow \forall A\_27x.$$
$$nonempty\ A\_27x \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_2$$
$$A\_27w\ A\_27x \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27x)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{((ty\_2Efc}$$
$$(62)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_$$
$$A\_27t\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{((ty\_2Efc}$$
$$(63)$$

Definition 70 *We define* c_2Emachine__ieee_2Efp16__to__float *to be* $\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ (ty\_2Efcp$

Definition 71 *We define* c_2Emachine__ieee_2Efp16__to__int *to be* $\lambda V0mode \in ty\_2Ebinary\_\_ieee\_2Eroundi$

Definition 72 *We define* c_2Ewords_2Eword__lsl *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

Definition 73 *We define* c_2Ewords_2Eword__or *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1v$

Definition 74 *We define* c_2Ewords_2Eword__join *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A$

Definition 75 *We define* c_2Ewords_2Eword__concat *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0v \in (ty\_2Ef$

Definition 76 *We define* c_2Emachine__ieee_2Efloat__to__fp16 *to be* $\lambda V0x \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}). (\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{64}$$

Assume the following.

$$(\forall V0x \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))).((ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efp16\_\_to\_\_float}\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp16}\ V0x)) = V0x)) \tag{65}$$

Theorem 1

$$((\forall V0mode \in ty\_2Ebinary\_\_ieee\_2Erounding.(\forall V1a \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))).((ap\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efp16\_\_to\_\_int}\ V0mode)\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp16}\ V1a)) = (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_to\_\_int}\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))\ V0mode)\ V1a)))) \wedge (\forall V2mode \in ty\_2Ebinary\_\_ieee\_2Erounding. (\forall V3a \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efp16\_\_to\_\_int}\ V2mode)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))))\ V3a)) = (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_to\_\_int}\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))\ V2mode)\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efp16\_\_to\_\_float}\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))))\ V3a)))))))$$