

thm\_2Emachine\_\_ieee\_2Efp32\_\_isSignallingNan  
(TMSE2QVG4Bftu7Z7j35AjA88jDYF5hWB29t)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2E$

**Definition 8** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2E$

Let  $c\_Earithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 10** We define  $c\_Ebit\_EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_Earithmetic\_EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 11** We define  $c\_Ebit\_EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebit\_EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 13** We define  $c\_Ebit\_EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Let  $ty\_2Efc\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efc\_2Efinite\_image A0) \quad (11)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (12)$$

Let  $c\_Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Ebool\_2Ethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (13)$$

Let  $c\_Efc\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Efc\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (14)$$

**Definition 14** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E.21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 15** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 17** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(ap (c\_2Emin\_2E\_3D\_3D\_3E V2t) c\_2Ebool\_2E\_7E))))$

**Definition 18** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A.\lambda y.p (ap P y)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 19** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) P)))$

**Definition 20** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Emin\_2E\_40 V0m) V1n)$

**Definition 21** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E\_2F\_5C A\_27a) P))$

**Definition 22** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(ap (c\_2Emin\_2E\_40 A\_27a) (ty\_2Enum\_2Enum A\_27a))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart A0 A1) \quad (15)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart A\_27a A\_27b \in ((A\_27a)^{(ty\_2Efcp\_2Efinite\_image A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)} \quad (16)$$

**Definition 23** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b)$

**Definition 24** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a)^{(ty\_2Enum\_2Enum)}) . (ap (c\_2Emin\_2E\_40 V0g) A\_27b)$

**Definition 25** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2Efcp\_2EFCP A\_27a) V0n)$

Let  $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efloat A0 A1) \quad (17)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand A\_27t A\_27w \in ((ty\_2Efcp\_2Ecart 2 A\_27t)^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)}) \quad (18)$$

**Definition 26** We define  $c\_2Ewords\_2Eword\_msb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a).(ap (c\_2Emin\_2E\_40 V0w) A\_27a)$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \quad (19)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \quad (20)$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealx\_2Ereal}) \quad (21)$$

**Definition 27** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 28** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Eboo$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (22)$$

**Definition 29** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcpx\_2Ecart\ 2\ A\_27a). (ap (ap ($

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (23)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (24)$$

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Ereal}) \quad (25)$$

**Definition 30** We define  $c\_2Erealx\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealx\_2Ereal. (ap (c\_2Emin\_2E40\ ($

Let  $c\_2Erealx\_2Etreall\_inv : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreall\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (26)$$

Let  $c\_2Erealx\_2Etreall\_eq : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreall\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (27)$$

Let  $c\_2Erealx\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_ABS\_CLASS \in (ty\_2Erealx\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (28)$$

**Definition 31** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)$

**Definition 32** We define  $c\_Erealax\_Einv$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal\_ABS)$

Let  $c\_Erealax\_Etrealmul : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealmul \in (((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal))^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)} \quad (29)$$

**Definition 33** We define  $c\_Erealax\_Ereal\_mul$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal.$

**Definition 34** We define  $c\_Ereal\_E\_2F$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal.$

Let  $c\_Erealax\_Etrealmul : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealmul \in (((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal))^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)} \quad (30)$$

**Definition 35** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal.$

Let  $c\_Ewords\_EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Ewords\_EINT\_MAX\ A\_27a \in (ty\_Eenum\_Eenum^{(ty\_Ebool\_Eitself\ A\_27a)}) \quad (31)$$

Let  $c\_Ebinary\_ieee\_Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_Exponent\ A\_27t\ A\_27w \in ((ty\_Efc\_Ecart\ 2\ A\_27w)^{(ty\_Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)}) \quad (32)$$

Let  $ty\_Eone\_Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_Eone\_Eone \quad (33)$$

Let  $c\_Ebinary\_ieee\_Efloat\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_Sign\ A\_27t\ A\_27w \in ((ty\_Efc\_Efloat\ 2\ ty\_Eone\_Eone)^{(ty\_Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)}) \quad (34)$$

Let  $c\_Erealax\_Etrealmul : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealmul \in ((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal))^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)} \quad (35)$$

**Definition 36** We define  $c\_Erealax\_Ereal\_neg$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal\_ABS)$

**Definition 37** We define  $c\_Ebinary\_ieee\_Efloat\_to\_real$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_Ebina$

Let  $ty\_2Ebinary\_ieee\_2Efloat\_value : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_value \quad (36)$$

Let  $c\_2Ebinary\_ieee\_2Efloat : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat \in (ty\_2Ebinary\_ieee\_2Efloat\_value^{ty\_2Erealax\_2Ereal}) \quad (37)$$

Let  $c\_2Ebinary\_ieee\_2ENaN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ENaN \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (38)$$

Let  $c\_2Ebinary\_ieee\_2EInfinity : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EInfinity \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (39)$$

Let  $c\_2Ewords\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (40)$$

**Definition 38** We define  $c\_2Ewords\_2Eword\_T$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ (ap\ (c\_2Ew$

**Definition 39** We define  $c\_2Ebinary\_ieee\_2Efloat\_value$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0x \in (ty\_2Ebinary\_$

Let  $c\_2Ebinary\_ieee\_2Efloat\_value\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_value\_CASE\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_ieee\_2Efloat\_value} \quad (41)$$

**Definition 40** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_nan$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0x \in (ty\_2Ebinary\_$

**Definition 41** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_signalling$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0x \in (ty\_2E$

**Definition 42** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1$

Let  $ty\_2EfcP\_2Ebit0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2EfcP\_2Ebit0\ A0) \quad (42)$$

Let  $ty\_2EfcP\_2Ebit1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2EfcP\_2Ebit1\ A0) \quad (43)$$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (44)$$

**Definition 43** We define  $c\_2Earithmetic\_2EMIN$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2E$

**Definition 44** We define `c_Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21\ 2) (\lambda V2t \in 2.$

**Definition 45** We define `c_Earithmic_2E_3C_3D` to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 46** We define `c_Ewords_2Eword_bits` to be  $\lambda A\_27a : \iota.\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.$

**Definition 47** We define `c_Ewords_2Ew2w` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a\ 27b).$

**Definition 48** We define `c_Ewords_2Eword_extract` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0h \in ty\_2Enum\_2Enum.$

**Definition 49** We define `c_Ecombin_2EK` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Let `c_Ebinary_ieee_2Efloat_Significand_fupd` :  $\iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27v.nonempty\ A\_27v \Rightarrow \\ & \quad nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_2Efloat\_Significand\_fupd\ A\_27t\ A\_27u\ A\_27v\ A\_27w \in \\ & \quad ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27u\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \end{aligned} \quad (45)$$

Let `c_Ebinary_ieee_2Efloat_Exponent_fupd` :  $\iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow \forall A\_27x.nonempty\ A\_27x \Rightarrow \\ & \quad nonempty\ A\_27y \Rightarrow c\_Ebinary\_ieee\_2Efloat\_Exponent\_fupd\ A\_27t\ A\_27w\ A\_27x\ A\_27y \in \\ & \quad ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27x)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \end{aligned} \quad (46)$$

Let `c_Ebinary_ieee_2Efloat_Sign_fupd` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_2Efloat\_Sign\_fupd\ A\_27t\ A\_27w \in \\ & \quad ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \end{aligned} \quad (47)$$

**Definition 50** We define `c_Emachine_ieee_2Efp32_to_float` to be  $\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ (ty\_2EfcP\_2Ecart\ 2\ 27a\ 27b)).$

**Definition 51** We define `c_Emachine_ieee_2Efp32_isSignallingNan` to be  $(ap (ap (c\_Ecombin_2Eo (ty\_2EfcP\_2Ecart\ 2\ 27a\ 27b)).$

Let `ty_2Esum_2Esum` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \end{aligned} \quad (48)$$

**Definition 52** We define `c_Ewords_2Eword_lsl` to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).\lambda V1l \in ty\_2Enum\_2Enum.$

**Definition 53** We define `c_Ewords_2Eword_or` to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).\lambda V1l \in ty\_2Enum\_2Enum.$

**Definition 54** We define `c_Ebool_2ELET` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27a.V0f))$

**Definition 55** We define `c_2Ewords_2Eword_join` to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ 2)$

**Definition 56** We define `c_2Ewords_2Eword_concat` to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ 2)$

**Definition 57** We define `c_2Emachine_ieee_2Efloat_to_fp32` to be  $\lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ (t))$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}). \\ & (\forall V2x \in A.27c.((ap\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A.27c\ A.27b\ A.27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ (ty\_2Efc\_2Ebit1\ ( \\ & ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone)))) \\ & (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))))). \\ & ((ap\ c\_2Emachine\_ieee\_2Efp32\_to\_float\ (ap\ c\_2Emachine\_ieee\_2Efloat\_to\_fp32 \\ & V0x)) = V0x)) \end{aligned} \quad (50)$$

**Theorem 1**

$$\begin{aligned} & ((\forall V0a \in (ty\_2Ebinary\_ieee\_2Efloat\ (ty\_2Efc\_2Ebit1 \\ & (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone)))) \\ & (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))))). \\ & ((p\ (ap\ c\_2Emachine\_ieee\_2Efp32\_isSignallingNan\ (ap\ c\_2Emachine\_ieee\_2Efloat\_to\_fp32 \\ & V0a))) \Leftrightarrow (p\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_is\_signalling\ (ty\_2Efc\_2Ebit1 \\ & (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone)))) \\ & (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone)))) \\ & V0a)))) \wedge (\forall V1a \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Emachine\_ieee\_2Efp32\_isSignallingNan \\ & (ap\ (c\_2Ewords\_2En2w\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0 \\ & (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))))))\ V1a))) \Leftrightarrow \\ & (p\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_is\_signalling\ (ty\_2Efc\_2Ebit1 \\ & (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone)))) \\ & (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone)))) \\ & (ap\ c\_2Emachine\_ieee\_2Efp32\_to\_float\ (ap\ (c\_2Ewords\_2En2w \\ & (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0 \\ & (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))))))\ V1a)))))) \end{aligned}$$