

thm_2Emachine_ieee_2Efp32_isSubnormal
(TMYuVzM-
CMzS5Ea9Zeo1biDChr6hzNjVDmoX)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ c_2Enum_2ESUC_REP\ (c_2Enum_2E0\ m)))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 10 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 11 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 13 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efc_2Efinite_image A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efc_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 14 We define $c_Ebool_2E_21_2$ to be $(ap (c_Ebool_2E_21_2) (\lambda V0t \in 2.V0t))$.

Definition 15 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 16 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_21_2))$

Definition 17 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21_2) (\lambda V2t \in 2.V2t))))$

Definition 18 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 19 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40 A_27a))))$

Definition 20 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_Ebool_2E_2F_5C A_27a))))$

Definition 22 We define $c_Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40 (A_27a^{ty_2Enum_2Enum})))$

Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efc_2Ecart A0 A1) \quad (15)$$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efc_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image A_27b)})^{(ty_2Efc_2Ecart A_27a A_27b)}) \quad (16)$$

Definition 23 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart A_27a A_27b)$

Definition 24 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efc_2Efc_index A_27a A_27b g)))$

Definition 25 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efc_2EFCP A_27a V0n))$

Let $ty_2Eb_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Eb_2Efloat A0 A1) \quad (17)$$

Let $c_2Eb_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Eb_2Efloat_Significand A_27t A_27w \in ((ty_2Efc_2Ecart 2 A_27t)^{(ty_2Eb_2Efloat A_27t A_27w)}) \quad (18)$$

Let $c_2Eb_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Eb_2Efloat_Exponent A_27t A_27w \in ((ty_2Efc_2Ecart 2 A_27w)^{(ty_2Eb_2Efloat A_27t A_27w)}) \quad (19)$$

Definition 26 We define $c_Ebinary_ieee_Efloat_is_subnormal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Eone_2Eone$

Definition 27 We define $c_Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (20)$$

Let $ty_2EfcP_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit0\ A0) \quad (21)$$

Let $ty_2EfcP_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit1\ A0) \quad (22)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (23)$$

Definition 28 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 29 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 30 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 31 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 32 We define $c_2Ewords_2Eword_bits$ to be $\lambda A_27a : \iota.\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum$

Definition 33 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})_{ty_2Enum_2Enum}) \quad (24)$$

Definition 34 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap\ (ap\ c$

Definition 35 We define $c_2Ewords_2Ew2w$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a$

Definition 36 We define $c_2Ewords_2Eword_extract$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0h \in ty_2Enum_2Enum$

Definition 37 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Let $c_2Ebinary_ieee_2Efloat_Significand_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27v. \\ & nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand_fupd \\ & A_27t\ A_27u\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27u\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (25)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x. \\ & nonempty\ A_27x \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A_27t \\ & A_27w\ A_27x \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Efloat_Exponent_fupd\ A_27t\ A_27w)})^{(ty_2Efloat_Exponent_fupd\ A_27t\ A_27x)} \end{aligned} \quad (26)$$

Let $c_2Ebinary_ieee_2Efloat_Sign_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign_fupd\ A_27t \\ & A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Efloat_Sign_fupd\ A_27t\ A_27w)})^{(ty_2Efloat_Sign_fupd\ A_27t\ A_27w)} \end{aligned} \quad (27)$$

Definition 38 We define $c_2Emachine_ieee_2Efp32_to_float$ to be $\lambda V0w \in (ty_2Efloat_Exponent_fupd\ 2\ (ty_2Efloat_Sign_fupd\ 2\ V0w))$

Definition 39 We define $c_2Emachine_ieee_2Efp32_isSubnormal$ to be $(ap\ (ap\ (c_2Ecombin_2Eo\ (ty_2Efloat_Exponent_fupd\ 2\ V0w))\ V0w))\ V0w$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \end{aligned} \quad (28)$$

Definition 40 We define $c_2Ewords_2Eword_lsl$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efloat_Exponent_fupd\ 2\ A_27a).\lambda V1x \in (ty_2Efloat_Sign_fupd\ 2\ V0w).V1x$

Definition 41 We define $c_2Ewords_2Eword_or$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efloat_Exponent_fupd\ 2\ A_27a).\lambda V1x \in (ty_2Efloat_Sign_fupd\ 2\ V0w).V1x$

Definition 42 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b.V0f\ V1x))$

Definition 43 We define $c_2Ewords_2Eword_join$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w \in (ty_2Efloat_Exponent_fupd\ 2\ A_27a).\lambda V1x \in (ty_2Efloat_Sign_fupd\ 2\ V0w).V1x$

Definition 44 We define $c_2Ewords_2Eword_concat$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0w \in (ty_2Efloat_Exponent_fupd\ 2\ A_27a).\lambda V1x \in (ty_2Efloat_Sign_fupd\ 2\ V0w).V1x$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign\ A_27t \\ & A_27t\ A_27w \in ((ty_2Efloat_Exponent_fupd\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Efloat_Sign\ A_27t\ A_27w)} \end{aligned} \quad (29)$$

Definition 45 We define $c_2Emachine_ieee_2Efloat_to_fp32$ to be $\lambda V0x \in (ty_2Ebinary_ieee_2Efloat\ (ty_2Efloat_Exponent_fupd\ 2\ V0x)\ (ty_2Efloat_Sign\ 2\ V0x))$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27c^{A_27a}). \\ & (\forall V2x \in A_27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Ebinary_ieee_2Efloat (ty_2Efc_2Ebit1 (\\
& ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))))). \\
& ((ap c_2Emachine_ieee_2Efp32_to_float (ap c_2Emachine_ieee_2Efloat_to_fp32 \\
& V0x)) = V0x))
\end{aligned} \tag{31}$$

Theorem 1

$$\begin{aligned}
& ((\forall V0a \in (ty_2Ebinary_ieee_2Efloat (ty_2Efc_2Ebit1 \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))))). \\
& ((p (ap c_2Emachine_ieee_2Efp32_isSubnormal (ap c_2Emachine_ieee_2Efloat_to_fp32 \\
& V0a))) \Leftrightarrow (p (ap (c_2Ebinary_ieee_2Efloat_is_subnormal (ty_2Efc_2Ebit1 \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) \\
& V0a)))) \wedge (\forall V1a \in ty_2Enum_2Enum. ((p (ap c_2Emachine_ieee_2Efp32_isSubnormal \\
& (ap (c_2Ewords_2En2w (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) V1a))) \Leftrightarrow \\
& (p (ap (c_2Ebinary_ieee_2Efloat_is_subnormal (ty_2Efc_2Ebit1 \\
& (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) \\
& (ap c_2Emachine_ieee_2Efp32_to_float (ap (c_2Ewords_2En2w \\
& (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) V1a))))))
\end{aligned}$$