

thm\_2Emachine\_\_ieee\_2Efp32\_\_negate  
(TMFowgSazTSMYT55TddAK3gQicj6sHa6N9u)

October 26, 2020

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2E$

**Definition 8** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2E$

Let  $c\_Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 10** We define  $c\_Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 11** We define  $c\_Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 13** We define  $c\_Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Let  $ty\_2EfcP\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2EfcP\_2Efinite\_image A0) \quad (11)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (12)$$

Let  $c\_Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Ebool\_2Ethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (13)$$

Let  $c\_2EfcP\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2EfcP\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (14)$$

**Definition 14** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E.21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 17** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(ap (c\_2Emin\_2E\_3D\_3D\_3E V2t) c\_2Ebool\_2E\_7E))))$

**Definition 18** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A.\lambda y.p (ap P y)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 19** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) P)))$

**Definition 20** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Emin\_2E\_40 A\_27a) m)$

**Definition 21** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E\_2F\_5C A\_27a) P))$

**Definition 22** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(ap (c\_2Emin\_2E\_40 A\_27a) (ty\_2Enum\_2Enum A\_27a))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart A0 A1) \quad (15)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart A\_27a A\_27b \in ((A\_27a)^{(ty\_2Efcp\_2Efinite\_image A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)} \quad (16)$$

**Definition 23** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b)$

**Definition 24** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a)^{(ty\_2Enum\_2Enum)})^{(ap (c\_2Efcp\_2EFCP A\_27a) g)}$

**Definition 25** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2Efcp\_2EFCP A\_27a) n)$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \quad (17)$$

Let  $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efloat A0 A1) \quad (18)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign A\_27t A\_27w \in ((ty\_2Efcp\_2Ecart 2 ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)}) \quad (19)$$



Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd\ A\_27t\ A\_27u\ A\_27w \in \\ & ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27u\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \end{aligned} \quad (25)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow \forall A\_27x.nonempty\ A\_27x \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd\ A\_27t\ A\_27w\ A\_27x \in \\ & (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27x)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \end{aligned} \quad (26)$$

**Definition 39** We define  $c\_2Emachine\_ieee\_2Efp32\_to\_float$  to be  $\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ (ty\_2Efc\_2Ecart\ 2\ A\_27a))$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\ A\_27t\ A\_27w \in \\ & ((ty\_2Efc\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \end{aligned} \quad (27)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent\ A\_27t\ A\_27w \in \\ & ((ty\_2Efc\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \end{aligned} \quad (28)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \end{aligned} \quad (29)$$

**Definition 40** We define  $c\_2Ewords\_2Eword\_lsl$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1x \in (ty\_2Efc\_2Ecart\ 2\ A\_27a)$

**Definition 41** We define  $c\_2Ewords\_2Eword\_or$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1u \in (ty\_2Efc\_2Ecart\ 2\ A\_27a)$

**Definition 42** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27b))$

**Definition 43** We define  $c\_2Ewords\_2Eword\_join$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a)$

**Definition 44** We define  $c\_2Ewords\_2Eword\_concat$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a)$

**Definition 45** We define  $c\_2Emachine\_ieee\_2Efloat\_to\_fp32$  to be  $\lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ A\_27a)$

**Definition 46** We define  $c\_2Emachine\_ieee\_2Efp32\_negate$  to be  $(ap\ (ap\ (c\_2Ecombin\_2Eo\ (ty\_2Efc\_2Ecart\ 2\ A\_27a))\ (ty\_2Efc\_2Ecart\ 2\ A\_27a))\ (ty\_2Efc\_2Ecart\ 2\ A\_27a))$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \forall A\_27c. \\
& \text{nonempty } A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27a^{A\_27c}). \\
& (\forall V2x \in A\_27c. ((\text{ap } (\text{ap } (\text{ap } (\text{c.2Ecombin\_2Eo } A\_27c } A\_27b } A\_27a) \\
& V0f) V1g) V2x) = (\text{ap } V0f (\text{ap } V1g V2x))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (\text{ty\_2Ebinary\_ieee\_2Efloat } (\text{ty\_2Efc\_2Ebit1 } ( \\
& \text{ty\_2Efc\_2Ebit1 } (\text{ty\_2Efc\_2Ebit1 } (\text{ty\_2Efc\_2Ebit0 } \text{ty\_2Eone\_2Eone})))) \\
& (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } \text{ty\_2Eone\_2Eone}))))). \\
& ((\text{ap } \text{c.2Emachine\_ieee\_2Efp32\_to\_float } (\text{ap } \text{c.2Emachine\_ieee\_2Efloat\_to\_fp32} \\
& V0x)) = V0x))
\end{aligned} \tag{31}$$

**Theorem 1**

$$\begin{aligned}
& ((\forall V0a \in (\text{ty\_2Ebinary\_ieee\_2Efloat } (\text{ty\_2Efc\_2Ebit1 } \\
& (\text{ty\_2Efc\_2Ebit1 } (\text{ty\_2Efc\_2Ebit1 } (\text{ty\_2Efc\_2Ebit0 } \text{ty\_2Eone\_2Eone})))) \\
& (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } \text{ty\_2Eone\_2Eone}))))). \\
& ((\text{ap } \text{c.2Emachine\_ieee\_2Efp32\_negate } (\text{ap } \text{c.2Emachine\_ieee\_2Efloat\_to\_fp32} \\
& V0a)) = (\text{ap } \text{c.2Emachine\_ieee\_2Efloat\_to\_fp32 } (\text{ap } (\text{c.2Ebinary\_ieee\_2Efloat\_negate} \\
& (\text{ty\_2Efc\_2Ebit1 } (\text{ty\_2Efc\_2Ebit1 } (\text{ty\_2Efc\_2Ebit1 } (\text{ty\_2Efc\_2Ebit0 } \\
& \text{ty\_2Eone\_2Eone})))) (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } \\
& \text{ty\_2Eone\_2Eone})))) V0a)))) \wedge (\forall V1a \in \text{ty\_2Enum\_2Enum.} (( \\
& \text{ap } \text{c.2Emachine\_ieee\_2Efp32\_negate } (\text{ap } (\text{c.2Ewords\_2En2w } (\text{ty\_2Efc\_2Ebit0} \\
& (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0} \\
& \text{ty\_2Eone\_2Eone})))))) V1a)) = (\text{ap } \text{c.2Emachine\_ieee\_2Efloat\_to\_fp32} \\
& (\text{ap } (\text{c.2Ebinary\_ieee\_2Efloat\_negate } (\text{ty\_2Efc\_2Ebit1 } (\text{ty\_2Efc\_2Ebit1} \\
& (\text{ty\_2Efc\_2Ebit1 } (\text{ty\_2Efc\_2Ebit0 } \text{ty\_2Eone\_2Eone})))) (\text{ty\_2Efc\_2Ebit0} \\
& (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } \text{ty\_2Eone\_2Eone})))) (\text{ap } \text{c.2Emachine\_ieee\_2Efp32\_to\_float} \\
& (\text{ap } (\text{c.2Ewords\_2En2w } (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0} \\
& (\text{ty\_2Efc\_2Ebit0 } (\text{ty\_2Efc\_2Ebit0 } \text{ty\_2Eone\_2Eone})))))) V1a))))))
\end{aligned}$$