

thm\_2Emachine\_ieee\_2Efp32\_sqrt  
(TMT29kbsmTdqcyS9gsAf4TXaNfyKiHncJQ7)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1P \in 2.V1P))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{2^m}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2EEXP$

**Definition 8** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2EEXP$

Let  $c\_Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (7)$$

Let  $c\_Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (8)$$

**Definition 10** We define  $c\_Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

Let  $c\_Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (10)$$

**Definition 11** We define  $c\_Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 13** We define  $c\_Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Let  $ty\_2EfcP\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2EfcP\_2Efinite\_image A0) \quad (11)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (12)$$

Let  $c\_Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Ebool\_2Ethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (13)$$

Let  $c\_2EfcP\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2EfcP\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (14)$$

**Definition 14** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E.21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 17** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(ap (c\_2Emin\_2E\_3D\_3D\_3E V2t) c\_2Ebool\_2E\_7E))))$

**Definition 18** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A.\lambda y.y \in A))$  of type  $\iota \Rightarrow \iota$ .

**Definition 19** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) P)))$

**Definition 20** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Emin\_2E\_40 V0m) V1n)$

**Definition 21** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E\_2F\_5C A\_27a) P))$

**Definition 22** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(ap (c\_2Emin\_2E\_40 A\_27a) (ty\_2Enum\_2Enum A\_27a))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart A0 A1) \quad (15)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_image A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \quad (16)$$

**Definition 23** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b)$

**Definition 24** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap (c\_2Efcp\_2Efcp\_index A\_27a A\_27b) g))$

**Definition 25** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2Efcp\_2EFCP A\_27a) V0n)$

Let  $ty\_2Ebinary\_ieee\_2Efp\_op : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efp\_op A0 A1) \quad (17)$$

Let  $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efloat A0 A1) \quad (18)$$

Let  $ty\_2Ebinary\_ieee\_2Errounding : \iota$  be given. Assume the following.

$$nonempty ty\_2Ebinary\_ieee\_2Errounding \quad (19)$$

Let  $c\_2Ebinary\_ieee\_2EFP\_Sqrt : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2EFP\_Sqrt\ A\_27t\ A\_27w \in (((ty\_2Ebinary\_ieee\_2Efp\_op\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})_{ty\_2Ebin}) \quad (20)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\ A\_27t\ A\_27w \in ((ty\_2Efc\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (21)$$

**Definition 26** We define  $c\_2Ewords\_2Eword\_msb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(ap$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \quad (22)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \quad (23)$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})_{ty\_2Erealx\_2Ereal}) \quad (24)$$

**Definition 27** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap$

**Definition 28** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebool$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})_{ty\_2Enum\_2Enum}) \quad (25)$$

**Definition 29** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (26)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (27)$$

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{ty\_2Erealx\_2Ereal}) \quad (28)$$

**Definition 30** We define  $c\_Erealax\_Ereal\_REP$  to be  $\lambda V0a \in ty\_Erealax\_Ereal.(ap (c\_Emin\_E40 (t$   
Let  $c\_Erealax\_Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_inv \in ((ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal) \quad (29)$$

Let  $c\_Erealax\_Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_eq \in ((2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)} (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) \quad (30)$$

Let  $c\_Erealax\_Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_ABS\_CLASS \in (ty\_Erealax\_Ereal)^{(2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)} (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal))} \quad (31)$$

**Definition 31** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)$

**Definition 32** We define  $c\_Erealax\_Einv$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap c\_Erealax\_Ereal\_ABS$

Let  $c\_Erealax\_Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_mul \in (((ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal) \quad (32)$$

**Definition 33** We define  $c\_Erealax\_Ereal\_mul$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal.$

**Definition 34** We define  $c\_Ereal\_E2F$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal.($

Let  $c\_Erealax\_Etrealm\_add : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_add \in (((ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal) \quad (33)$$

**Definition 35** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal.$

Let  $c\_Ewords\_EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Ewords\_EINT\_MAX A\_27a \in (ty\_Eenum\_Eenum)^{(ty\_Ebool\_Eitself A\_27a)} \quad (34)$$

Let  $c\_Ebinary\_ieee\_Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_Exponent A\_27t A\_27w \in ((ty\_Efc\_Ecart 2 A\_27w)^{(ty\_Ebinary\_ieee\_Efloat A\_27t A\_27w)}) \quad (35)$$

Let  $ty\_Eone\_Eone : \iota$  be given. Assume the following.

$$nonempty ty\_Eone\_Eone \quad (36)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (37)$$

Let  $c\_2Erealax\_2Etreax\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreax\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (38)$$

**Definition 36** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

**Definition 37** We define  $c\_2Ebinary\_ieee\_2Efloat\_to\_real$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary$

Let  $ty\_2Ebinary\_ieee\_2Efloat\_value : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_value \quad (39)$$

Let  $c\_2Ebinary\_ieee\_2Efloat : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat \in (ty\_2Ebinary\_ieee\_2Efloat\_value^{ty\_2Erealax\_2Ereal}) \quad (40)$$

Let  $c\_2Ebinary\_ieee\_2ENaN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ENaN \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (41)$$

Let  $c\_2Ebinary\_ieee\_2EInfinity : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EInfinity \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (42)$$

Let  $c\_2Ewords\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_MAX\ A\_27a \in (ty\_2Eenum\_2Eenum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (43)$$

**Definition 38** We define  $c\_2Ewords\_2Eword\_T$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Ewords\_2Een2w\ A\_27a)\ (ap\ (c\_2Ew$

**Definition 39** We define  $c\_2Ebinary\_ieee\_2Efloat\_value$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary$

Let  $c\_2Ebinary\_ieee\_2Efloat\_value\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_value\_CASE\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_ieee\_2Efloat\_value} \quad (44)$$

**Definition 40** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_nan$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary$

**Definition 41** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_signalling$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary$

**Definition 42** We define `c2Ebool_2ELET` to be  $\lambda A.27a : \iota. \lambda A.27b : \iota. (\lambda V0f \in (A.27b^{A.27a}). (\lambda V1x \in A.27b. V0x \in A.27a)))$

**Definition 43** We define `c2Ebinary_ieee_2Efloat_some_qnan` to be  $\lambda A.27t : \iota. \lambda A.27w : \iota. \lambda V0fp\_op \in (ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags})$

Let `ty_2Ebinary_ieee_2Efloat_some_qnan` be given. Assume the following.

$$\text{nonempty } ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags} \quad (45)$$

Let `c2Ebool_2EARB` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a. \text{nonempty } A.27a \Rightarrow c\_2Ebool\_2EARB \ A.27a \in A.27a \quad (46)$$

**Definition 44** We define `c2Ecombin_2EK` to be  $\lambda A.27a : \iota. \lambda A.27b : \iota. (\lambda V0x \in A.27a. (\lambda V1y \in A.27b. V0x \in A.27a)))$

Let `c2Ebinary_ieee_2Efloat_some_qnan_Underflow_AfterRounding_fupd` :  $\iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat\_some\_qnan\_Underflow\_AfterRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags})^{ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags}})^{(2^2)} \quad (47)$$

Let `c2Ebinary_ieee_2Efloat_some_qnan_Underflow_BeforeRounding_fupd` :  $\iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat\_some\_qnan\_Underflow\_BeforeRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags})^{ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags}})^{(2^2)} \quad (48)$$

Let `c2Ebinary_ieee_2Efloat_some_qnan_Precision_fupd` :  $\iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat\_some\_qnan\_Precision\_fupd \in ((ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags})^{ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags}})^{(2^2)} \quad (49)$$

Let `c2Ebinary_ieee_2Efloat_some_qnan_Overflow_fupd` :  $\iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat\_some\_qnan\_Overflow\_fupd \in ((ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags})^{ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags}})^{(2^2)} \quad (50)$$

Let `c2Ebinary_ieee_2Efloat_some_qnan_InvalidOp_fupd` :  $\iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat\_some\_qnan\_InvalidOp\_fupd \in ((ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags})^{ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags}})^{(2^2)} \quad (51)$$

Let `c2Ebinary_ieee_2Efloat_some_qnan_DivideByZero_fupd` :  $\iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat\_some\_qnan\_DivideByZero\_fupd \in ((ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags})^{ty\_2Ebinary\_ieee\_2Efloat\_some\_qnan \text{ flags}})^{(2^2)} \quad (52)$$

**Definition 45** We define `c2Ebinary_ieee_2Eclear_flags` to be  $(\text{ap } (\text{ap } c\_2Ebinary\_ieee\_2Efloat\_some\_qnan\_DivideByZero\_fupd))$

**Definition 46** We define `c2Ebinary_ieee_2Einvalidop_flags` to be  $(\text{ap } (\text{ap } c\_2Ebinary\_ieee\_2Efloat\_some\_qnan\_InvalidOp\_fupd))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}})$$
(53)

**Definition 47** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ x\ y)$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0)$$
(54)

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a)$$
(55)

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
(56)

Let  $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})})$$
(57)

**Definition 48** We define  $c\_2Ebinary\_ieee\_2Echeck\_for\_signalling$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0l \in (ty\_2Ebinary\_ieee\_2Efloat\_plus\_infinity\ A\_27a\ A\_27b)$

Let  $c\_2Ebinary\_ieee\_2Efloat\_plus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_plus\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))})$$
(58)

Let  $c\_2Erealx\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})$$
(59)

**Definition 49** We define  $c\_2Erealx\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.\lambda V1T2 \in ty\_2Erealx\_2Ereal$

**Definition 50** We define  $c\_2Etransc\_2Eroot$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1x \in ty\_2Erealx\_2Ereal$

**Definition 51** We define  $c\_2Etransc\_2Esqrt$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.(ap\ (ap\ c\_2Etransc\_2Eroot\ x))$

**Definition 52** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.\lambda V1y \in ty\_2Erealx\_2Ereal$

**Definition 53** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECONJ\ x\ 0))))$



Let  $c\_2Ebinary\_ieee\_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Elargest \\ & A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (60)$$

**Definition 54** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_finite$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\_is\_finite\ A\_27t\ A\_27w)$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (61)$$

**Definition 55** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 56** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 57** We define  $c\_2Ebinary\_ieee\_2Eis\_closest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{(ty\_2Ebinary\_ieee\_2Efloat\_is\_finite\ A\_27a\ A\_27b)})$

**Definition 58** We define  $c\_2Ebinary\_ieee\_2Eclosest\_such$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Ebinary\_ieee\_2Efloat\_is\_finite\ A\_27a\ A\_27b)})$

**Definition 59** We define  $c\_2Ebinary\_ieee\_2Eclosest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Ebinary\_ieee\_2Eclosest\_such\ A\_27a\ A\_27b))$

Let  $c\_2Ebinary\_ieee\_2Efloat\_top : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_top \\ & A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (62)$$

**Definition 60** We define  $c\_2Ereal\_2Ereal\_gt$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ebinary\_ieee\_2Efloat\_bottom : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_bottom \\ & A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (63)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_minus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_minus\_infinity \\ & A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (64)$$

**Definition 61** We define  $c\_2Ereal\_2Ereal\_ge$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ebinary\_ieee\_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Ethreshold \\ & A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (65)$$

**Definition 62** We define  $c\_Ewords\_Eword\_lsb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(ap$

Let  $c\_Ebinary\_ieee\_Erounding2num : \iota$  be given. Assume the following.

$$c\_Ebinary\_ieee\_Erounding2num \in (ty\_2Enum\_2Enum^{ty\_2Ebinary\_ieee\_Erounding}) \quad (66)$$

**Definition 63** We define  $c\_Ebinary\_ieee\_Erounding\_CASE$  to be  $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Ebinary\_ieee\_2$

**Definition 64** We define  $c\_Ebinary\_ieee\_ERound$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinary\_ie$

Let  $c\_Ebinary\_ieee\_Efloat\_plus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_plus\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)})) \quad (67)$$

Let  $c\_Ebinary\_ieee\_Efloat\_minus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_minus\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)})) \quad (68)$$

**Definition 65** We define  $c\_Ebinary\_ieee\_Efloat\_is\_zero$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

**Definition 66** We define  $c\_Ebinary\_ieee\_Efloat\_round$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebin$

Let  $c\_Ewords\_2EINT\_MIN : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Ewords\_2EINT\_MIN\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (69)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (70)$$

Let  $c\_Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (71)$$

**Definition 67** We define  $c\_Ewords\_Eword\_2comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).$

**Definition 68** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 69** We define  $c\_Ewords\_2Enzcv$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1b \in ($

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (72)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (73)$$

**Definition 70** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})$

**Definition 71** We define  $c\_2Ewords\_2Eword\_ls$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).\lambda V1b$

**Definition 72** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_infinite$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

**Definition 73** We define  $c\_2Ebinary\_ieee\_2Efloat\_round\_with\_flags$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode$

**Definition 74** We define  $c\_2Ebinary\_ieee\_2Efloat\_sqrt$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinar$

**Definition 75** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let  $ty\_2EfcP\_2Ebit0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2EfcP\_2Ebit0\ A0) \quad (74)$$

Let  $ty\_2EfcP\_2Ebit1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2EfcP\_2Ebit1\ A0) \quad (75)$$

**Definition 76** We define  $c\_2Earithmetic\_2EMIN$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 77** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

**Definition 78** We define  $c\_2Ewords\_2Eword\_bits$  to be  $\lambda A\_27a : \iota.\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2$

**Definition 79** We define  $c\_2Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a$

**Definition 80** We define  $c\_2Ewords\_2Eword\_extract$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0h \in ty\_2Enum\_2Enum$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27v \\ nonempty\ A\_27v \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\_f \\ A\_27t\ A\_27u\ A\_27v \in (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27u\ A\_27v)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27v)}) \end{aligned} \quad (76)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow \forall A\_27x. \\ & nonempty\ A\_27x \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd\ A\_27t\ A\_27w\ A\_27x \in \\ & (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27x)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Efloat\ A\_27t\ A\_27x)}) \\ & \quad (77) \end{aligned}$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign\_fupd : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign\_fupd\ A\_27t\ A\_27w \in \\ & (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Efloat\ A\_27t\ A\_27w)}) \\ & \quad (78) \end{aligned}$$

**Definition 81** We define  $c\_2Emachine\_ieee\_2Efp32\_to\_float$  to be  $\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ (ty\_2Efc\_2Ebit1\ 2\ A\_27a))$ .

**Definition 82** We define  $c\_2Ewords\_2Eword\_lsl$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1v \in (ty\_2Efc\_2Ebit1\ 2\ A\_27a)$ .

**Definition 83** We define  $c\_2Ewords\_2Eword\_or$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1v \in (ty\_2Efc\_2Ebit1\ 2\ A\_27a)$ .

**Definition 84** We define  $c\_2Ewords\_2Eword\_join$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a)$ .

**Definition 85** We define  $c\_2Ewords\_2Eword\_concat$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0v \in (ty\_2Efc\_2Ebit1\ 2\ A\_27a)$ .

**Definition 86** We define  $c\_2Emachine\_ieee\_2Efloat\_to\_fp32$  to be  $\lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ (ty\_2Efloat\ A\_27t\ A\_27w))$ .

**Definition 87** We define  $c\_2Emachine\_ieee\_2Efp32\_sqrt$  to be  $\lambda V0mode \in ty\_2Ebinary\_ieee\_2Errounding$ .

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}). \\ & (\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27c\ A\_27b\ A\_27a)) \\ & \quad V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \\ & \quad (79) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ (ty\_2Efc\_2Ebit1\ 2\ A\_27a))) \\ & (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit1\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone)))) \\ & (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ (ty\_2Efc\_2Ebit0\ ty\_2Eone\_2Eone))))). \\ & ((ap\ c\_2Emachine\_ieee\_2Efp32\_to\_float\ (ap\ c\_2Emachine\_ieee\_2Efloat\_to\_fp32 \\ & \quad V0x)) = V0x) \\ & \quad (80) \end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& ((\forall V0mode \in ty\_2Ebinary\_ieee\_2Errounding. (\forall V1a \in \\
& (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit1 \\
& (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) (ty\_2Efc\_2Ebit0 \\
& (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))))). (ap \\
& (ap c\_2Emachine\_ieee\_2Efp32\_sqrt V0mode) (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp32 \\
& V1a)) = (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp32 (ap (c\_2Epair\_2ESND \\
& ty\_2Ebinary\_ieee\_2Eflags (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit1 \\
& (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) \\
& (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) \\
& (ap (ap (c\_2Ebinary\_ieee\_2Efloat\_sqrt (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit1 \\
& (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) (ty\_2Efc\_2Ebit0 \\
& (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) V0mode) \\
& V1a)))))) \wedge (\forall V2mode \in ty\_2Ebinary\_ieee\_2Errounding. ( \\
& \forall V3a \in ty\_2Enum\_2Enum. ((ap (ap c\_2Emachine\_ieee\_2Efp32\_sqrt \\
& V2mode) (ap (c\_2Ewords\_2En2w (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 \\
& (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))))) \\
& V3a)) = (ap c\_2Emachine\_ieee\_2Efloat\_to\_fp32 (ap (c\_2Epair\_2ESND \\
& ty\_2Ebinary\_ieee\_2Eflags (ty\_2Ebinary\_ieee\_2Efloat (ty\_2Efc\_2Ebit1 \\
& (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) \\
& (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) \\
& (ap (ap (c\_2Ebinary\_ieee\_2Efloat\_sqrt (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit1 \\
& (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) (ty\_2Efc\_2Ebit0 \\
& (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) V2mode) \\
& (ap c\_2Emachine\_ieee\_2Efp32\_to\_float (ap (c\_2Ewords\_2En2w \\
& (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit0 \\
& (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))))) V3a)))))))))
\end{aligned}$$