thm_2Emachine__ieee_2Efp64__add__with__flags (TMJNBPAXdT1MKpf9WyrK35yaEzM2dDof8UW)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 9 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 10 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Definition 11 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 12 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 13 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{11}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{12}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{13}$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{14}$$

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 16 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 17 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 18 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 19 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 20 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 21 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}$

Definition 22 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{15}$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \tag{16}$$

Definition 23 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Definition 24 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 25 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (\mathsf{c\_2Efcp\_2EFC}$

Let $ty\_2Ebinary\_\_ieee\_2Efp\_\_op : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_\_ieee\_2Efp\_\_op\ A0\ A1) \tag{17}$$

Let $ty\_2Ebinary\_\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A0\ A1) \tag{18}$$

Let $ty\_2Ebinary\_\_ieee\_2Erounding : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Erounding \tag{19}$$

Let $c\_2Ebinary\_\_ieee\_2EFP\_\_Add : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\ A\_27t\ A\_27w \in ((((ty\_2Ebinary\_\_ieee\_2Efp\_\_op\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2E} \tag{20}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{21}$$

Definition 26 *We define* c_2Ewords_2Eword__msb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{22}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{23}$$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{24}$$

Definition 27 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 28 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Eboo

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \tag{25}$$

Definition 29 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{26}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{27}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{28}$$

Definition 30 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 (t

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (29)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (30)$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \quad (31)$$

Definition 31 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 32 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (32)$$

Definition 33 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 34 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (33)$$

Definition 35 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Ewords\_2EINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_2} \quad (34)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (35)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (36)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{37}$$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{38}$$

Definition 36 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Definition 37 *We define* c_2Ebinary__ieee_2Efloat__to__real *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Let $ty\_2Ebinary\_\_ieee\_2Efloat\_\_value : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{39}$$

Let $c\_2Ebinary\_\_ieee\_2EFloat : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EFloat \in (ty\_2Ebinary\_\_ieee\_2Efloat\_\_value^{ty\_2Erealax\_2Ereal}) \tag{40}$$

Let $c\_2Ebinary\_\_ieee\_2ENaN : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2ENaN \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{41}$$

Let $c\_2Ebinary\_\_ieee\_2EInfinity : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EInfinity \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{42}$$

Let $c\_2Ewords\_2EUINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{43}$$

Definition 38 *We define* c_2Ewords_2Eword__T *to be* $\lambda A\_27a : \iota.(ap$ (c_2Ewords_2En2w A_27a) $(ap\ (c\_2Ew$

Definition 39 *We define* c_2Ebinary__ieee_2Efloat__value *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE\ A\_27a \in ((((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_\_ieee\_2Efloat\_\_value}) \tag{44}$$

Definition 40 *We define* c_2Ebinary__ieee_2Efloat__is__nan *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Definition 41 *We define* c_2Ebinary__ieee_2Efloat__is__signalling *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Eb$

Definition 42 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.(\lambda V0f\in(A\_27b^{A\_27a}).(\lambda V1x\in A\_27$

Definition 43 *We define* c_2Ebinary__ieee_2Efloat__some__qnan *to be* $\lambda A\_27t:\iota.\lambda A\_27w:\iota.\lambda V0fp\_\_op\in(ty$

Let $ty\_2Elist\_2Elist:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{45}$$

Let $c\_2Elist\_2ENIL:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2ENIL\ A\_27a\in(ty\_2Elist\_2Elist\ A\_27a) \tag{46}$$

Let $c\_2Elist\_2ECONS:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2ECONS\ A\_27a\in(((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{47}$$

Let $ty\_2Ebinary\_\_ieee\_2Eflags:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Eflags \tag{48}$$

Let $c\_2Ebool\_2EARB:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ebool\_2EARB\ A\_27a\in A\_27a \tag{49}$$

Definition 44 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.(\lambda V0x\in A\_27a.(\lambda V1y\in A\_27b.V0x)$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_Underflow\_\_AfterRounding\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_Underflow\_\_AfterRounding\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)}) \tag{50}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_Underflow\_\_BeforeRounding\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_Underflow\_\_BeforeRounding\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)}) \tag{51}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_Precision\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_Precision\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)}) \tag{52}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_Overflow\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_Overflow\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)}) \tag{53}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_InvalidOp\_\_fupd : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_InvalidOp\_\_fupd \in ((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)} \tag{54}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_DivideByZero\_\_fupd : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_DivideByZero\_\_fupd \in ((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags} \tag{55}$$

Definition 45 *We define* c_2Ebinary__ieee_2Eclear__flags *to be* $(ap\ (ap\ c\_2Ebinary\_\_ieee\_2Eflags\_\_DivideE$

Let $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{56}$$

Definition 46 *We define* c_2Ebinary__ieee_2Echeck__for__signalling *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0l \in (ty\_2E$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{57}$$

Definition 47 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 48 *We define* c_2Ebinary__ieee_2Einvalidop__flags *to be* $(ap\ (ap\ c\_2Ebinary\_\_ieee\_2Eflags\_\_Inv$

Let $c\_2Ebinary\_\_ieee\_2EroundTowardNegative : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EroundTowardNegative \in ty\_2Ebinary\_\_ieee\_2Erounding \tag{58}$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{59}$$

Definition 49 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 50 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 51 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ ($c_2Ebool_2ECONI

Let $c\_2Ebinary\_\_ieee\_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Elargest\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \tag{60}$$

Definition 52 *We define* c_2Ebinary__ieee_2Efloat__is__finite *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{61}$$

Definition 53 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Definition 54 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 55 *We define* c_2Ebinary__ieee_2Eis__closest *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{(ty\_2Ebinary\_\_ie}$

Definition 56 *We define* c_2Ebinary__ieee_2Eclosest__such *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Ebinary}$

Definition 57 *We define* c_2Ebinary__ieee_2Eclosest *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap$ (c_2Ebinary__ieee_2Eclos

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_top : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_top\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{62}$$

Definition 58 *We define* c_2Ereal_2Ereal__gt *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_bottom : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_bottom\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{63}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{64}$$

Definition 59 *We define* c_2Ereal_2Ereal__ge *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{65}$$

Let $c\_2Ebinary\_\_ieee\_2Ethreshold : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\_2Ethreshold\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \tag{66}$$

Definition 60 *We define* c_2Ewords_2Eword__lsb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Let $c\_2Ebinary\_\_ieee\_2Erounding2num : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Erounding2num \in (ty\_2Enum\_2Enum^{ty\_2Ebinary\_\_ieee\_2Erounding}) \quad (67)$$

Definition 61 *We define* c_2Ebinary__ieee_2Erounding__CASE *to be* $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Ebinary\_\_ieee\_2$

Definition 62 *We define* c_2Ebinary__ieee_2Eround *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinary\_\_ie$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (68)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (69)$$

Definition 63 *We define* c_2Ebinary__ieee_2Efloat__is__zero *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Definition 64 *We define* c_2Ebinary__ieee_2Efloat__round *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebina$

Let $c\_2Ewords\_2EINT\_\_MIN : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ewords\_2EINT\_\_MIN\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27} \quad (70)$$

Let $ty\_2Esum\_2Esum : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (71)$$

Let $c\_2Ewords\_2Edimword : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (72)$$

Definition 65 *We define* c_2Ewords_2Eword__2comp *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Definition 66 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 67 *We define* c_2Ewords_2Enzcv *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b \in ($

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{73}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{74}$$

Definition 68 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 69 *We define* c_2Ewords_2Eword__ls *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b$

Definition 70 *We define* c_2Ebinary__ieee_2Efloat__is__infinite *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

Definition 71 *We define* c_2Ebinary__ieee_2Efloat__round__with__flags *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode$

Definition 72 *We define* c_2Epair_2Epair__CASE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair\_$

Definition 73 *We define* c_2Ebinary__ieee_2Efloat__add *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinar$

Definition 74 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 75 *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 76 *We define* c_2Epair_2E_23_23 *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f \in (A\_2$

Let $ty\_2Efcp\_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Ebit0\ A0) \tag{75}$$

Let $ty\_2Efcp\_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Ebit1\ A0) \tag{76}$$

Definition 77 *We define* c_2Earithmetic_2EMIN *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 78 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 79 *We define* c_2Ewords_2Eword__bits *to be* $\lambda A\_27a : \iota.\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2$

Definition 80 *We define* c_2Ewords_2Ew2w *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27$

Definition 81 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Definition 82 *We define* c_2Ewords_2Eword__extract *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0h \in ty\_2Enum\_2Enum$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd : \iota{\Rightarrow}\iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27u.nonempty\ A\_27u{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_f\ A\_27t\ A\_27u\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27u\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})\tag{77}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd : \iota{\Rightarrow}\iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}\forall A\_27x.\ nonempty\ A\_27x{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_2\ A\_27w\ A\_27x \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27x)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{((ty\_2Efc}\tag{78}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\ A\_27t\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{((ty\_2Efc}\tag{79}$$

Definition 83 *We define* c_2Emachine__ieee_2Efp64__to__float *to be* $\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ (ty\_2Efc$

Definition 84 *We define* c_2Ewords_2Eword__lsl *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

Definition 85 *We define* c_2Ewords_2Eword__or *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1v$

Definition 86 *We define* c_2Ewords_2Eword__join *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2$

Definition 87 *We define* c_2Ewords_2Eword__concat *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0v \in (ty\_2Ef$

Definition 88 *We define* c_2Emachine__ieee_2Efloat__to__fp64 *to be* $\lambda V0x \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (t$

Definition 89 *We define* c_2Emachine__ieee_2Efp64__add__with__flags *to be* $\lambda V0mode \in ty\_2Ebinary\_\_ieee\_$

Assume the following.

$$(\forall V0x \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ ty\_2Eone\_2Eone))))) (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))).((ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efp64\_\_to\_\_float}\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp64}\ V0x)) = V0x))\tag{80}$$

Theorem 1

$$
\begin{gathered}
((\forall V0mode \in ty\_2Ebinary\_\_ieee\_2Erounding.(\forall V1b \in \\
(ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ ty\_2Eone\_2Eone))))) \\
(ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))). \\
(\forall V2a \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit0\ (\\
ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1 \\
ty\_2Eone\_2Eone))))) (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0 \\
ty\_2Eone\_2Eone)))).((ap\ (ap\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efp64\_\_add\_\_with\_\_flags} \\
V0mode)\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp64}\ V2a))\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp64} \\
V1b)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23}\ ty\_2Ebinary\_\_ieee\_2Eflags \\
(ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ ty\_2Eone\_2Eone))))) \\
(ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))) \\
ty\_2Ebinary\_\_ieee\_2Eflags\ (ty\_2Efcp\_2Ecart\ 2\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))))))) (\mathsf{c\_2Ecombin\_2EI}\ ty\_2Ebinary\_\_ieee\_2Eflags)) \\
\mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp64})\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_add} \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit1\ ty\_2Eone\_2Eone))))) (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1 \\
(ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))))\ V0mode)\ V2a)\ V1b))))))\wedge \\
((\forall V3mode \in ty\_2Ebinary\_\_ieee\_2Erounding.(\forall V4b \in \\
ty\_2Enum\_2Enum.(\forall V5a \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1 \\
ty\_2Eone\_2Eone))))) (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0 \\
ty\_2Eone\_2Eone)))).((ap\ (ap\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efp64\_\_add\_\_with\_\_flags} \\
V3mode)\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp64}\ V5a))\ (ap\ (\mathsf{c\_2Ewords\_2En2w} \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))))))\ V4b)) = \\
(ap\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23}\ ty\_2Ebinary\_\_ieee\_2Eflags\ (ty\_2Ebinary\_\_ieee\_2Efloat \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit1\ ty\_2Eone\_2Eone))))) (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1 \\
(ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))))\ ty\_2Ebinary\_\_ieee\_2Eflags \\
(ty\_2Efcp\_2Ecart\ 2\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))))))) \\
(\mathsf{c\_2Ecombin\_2EI}\ ty\_2Ebinary\_\_ieee\_2Eflags))\ \mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp64}) \\
(ap\ (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_add}\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1 \\
ty\_2Eone\_2Eone))))) (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0 \\
ty\_2Eone\_2Eone))))\ V3mode)\ V5a)\ (ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efp64\_\_to\_\_float} \\
(ap\ (\mathsf{c\_2Ewords\_2En2w}\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))))))) \\
V4b))))))))\wedge((\forall V6mode \in ty\_2Ebinary\_\_ieee\_2Erounding. \\
(\forall V7b \in (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit0\ (\\
ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1 \\
ty\_2Eone\_2Eone))))) (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0 \\
ty\_2Eone\_2Eone)))).(\forall V8a \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap \\
\mathsf{c\_2Emachine\_\_ieee\_2Efp64\_\_add\_\_with\_\_flags}\ V6mode)\ (ap\ (\mathsf{c\_2Ewords\_2En2w} \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))))))\ V8a)) \\
(ap\ \mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp64}\ V7b)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23} \\
ty\_2Ebinary\_\_ieee\_2Eflags\ (ty\_2Ebinary\_\_ieee\_2Efloat\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1 \\
ty\_2Eone\_2Eone))))) (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0 \\
ty\_2Eone\_2Eone))))\ ty\_2Ebinary\_\_ieee\_2Eflags\ (ty\_2Efcp\_2Ecart \\
2\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))))))))\ (\\
\mathsf{c\_2Ecombin\_2EI}\ ty\_2Ebinary\_\_ieee\_2Eflags))\ \mathsf{c\_2Emachine\_\_ieee\_2Efloat\_\_to\_\_fp64}) \\
(ap\ (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_add}\ (ty\_2Efcp\_2Ebit0 \\
(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit1
\end{gathered}
$$