

thm_2Emachine_ieee_2Efp64_isSignallingNan
 (TMJWbn-
 QvfE8UfWKM2yNdsfhguVejAaPDqjU)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Earithmic_2EZERO\ c_2Enum_2E0\ c_2Enum_2EREP_num\ c_2Enum_2ESUC_REP))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 10 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 11 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 13 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efc_2Efinite_image A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efc_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 14 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 15 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21\ 2))$

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Definition 18 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 19 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a))))$

Definition 20 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C A_27a))))$

Definition 22 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum})))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcp_2Ecart\ A0\ A1) \quad (15)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcp_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \quad (16)$$

Definition 23 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Definition 24 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efcp_2Efcp_index\ A_27a\ A_27b\ g)))$

Definition 25 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcp_2EFCP\ A_27a\ n))$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (17)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2Efcp_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (18)$$

Definition 26 We define $c_2Ewords_2Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a).(ap (c_2Eword_msb\ A_27a\ w))$

Definition 31 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)$

Definition 32 We define $c_Erealax_Einv$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal_ABS)$

Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal))^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)} \quad (29)$$

Definition 33 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Definition 34 We define $c_Ereal_E_2F$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.$

Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul : \iota \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal))^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)} \quad (30)$$

Definition 35 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Let $c_Ewords_EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ewords_EINT_MAX\ A_27a \in (ty_Eenum_Eenum^{(ty_Ebool_Eitself\ A_27a)}) \quad (31)$$

Let $c_Ebinary_ieee_Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_Exponent\ A_27t\ A_27w \in ((ty_Efc_Ecart\ 2\ A_27w)^{(ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)}) \quad (32)$$

Let $ty_Eone_Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_Eone_Eone \quad (33)$$

Let $c_Ebinary_ieee_Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_Sign\ A_27t\ A_27w \in ((ty_Efc_Efloat\ 2\ ty_Eone_Eone)^{(ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)}) \quad (34)$$

Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul : \iota \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal))^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)} \quad (35)$$

Definition 36 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal_ABS)$

Definition 37 We define $c_Ebinary_ieee_Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebina$

Let $ty_2Ebinary_ieee_2Efloat_value : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_value \quad (36)$$

Let $c_2Ebinary_ieee_2Efloat : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Efloat \in (ty_2Ebinary_ieee_2Efloat_value^{ty_2Erealax_2Ereal}) \quad (37)$$

Let $c_2Ebinary_ieee_2ENaN : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2ENaN \in ty_2Ebinary_ieee_2Efloat_value \quad (38)$$

Let $c_2Ebinary_ieee_2EInfinity : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EInfinity \in ty_2Ebinary_ieee_2Efloat_value \quad (39)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (40)$$

Definition 38 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ (c_2Ew$

Definition 39 We define $c_2Ebinary_ieee_2Efloat_value$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_$

Let $c_2Ebinary_ieee_2Efloat_value_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebinary_ieee_2Efloat_value_CASE\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebinary_ieee_2Efloat_value} \quad (41)$$

Definition 40 We define $c_2Ebinary_ieee_2Efloat_is_nan$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_$

Definition 41 We define $c_2Ebinary_ieee_2Efloat_is_signalling$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2E$

Definition 42 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $ty_2EfcP_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit0\ A0) \quad (42)$$

Let $ty_2EfcP_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit1\ A0) \quad (43)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (44)$$

Definition 43 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2E$

Definition 44 We define `c_Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21\ 2) (\lambda V2t \in 2.$

Definition 45 We define `c_Earithmic_2E_3C_3D` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 46 We define `c_Ewords_2Eword_bits` to be $\lambda A_27a : \iota.\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.$

Definition 47 We define `c_Ewords_2Ew2w` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a\ 2\ A_27b).$

Definition 48 We define `c_Ewords_2Eword_extract` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0h \in ty_2Enum_2Enum.$

Definition 49 We define `c_Ecombin_2EK` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Let `c_Ebinary_ieee_2Efloat_Significand_fupd` : $\iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27v.nonempty\ A_27v \Rightarrow \\ & \quad nonempty\ A_27w \Rightarrow c_Ebinary_ieee_2Efloat_Significand_fupd\ A_27t\ A_27u\ A_27v\ A_27w \in \\ & \quad ((ty_2Ebinary_ieee_2Efloat\ A_27u\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (45)$$

Let `c_Ebinary_ieee_2Efloat_Exponent_fupd` : $\iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x.nonempty\ A_27x \Rightarrow \\ & \quad nonempty\ A_27y \Rightarrow c_Ebinary_ieee_2Efloat_Exponent_fupd\ A_27t\ A_27w\ A_27x\ A_27y \in \\ & \quad ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (46)$$

Let `c_Ebinary_ieee_2Efloat_Sign_fupd` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_2Efloat_Sign_fupd\ A_27t\ A_27w \in \\ & \quad ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (47)$$

Definition 50 We define `c_Emachine_ieee_2Efp64_to_float` to be $\lambda V0w \in (ty_2Efc_2Ecart\ 2\ (ty_2Efc_2Ecart\ 2\ A_27a\ 2\ A_27b)).$

Definition 51 We define `c_Emachine_ieee_2Efp64_isSignallingNan` to be $(ap (ap (c_Ecombin_2Eo (ty_2Efc_2Ecart\ 2\ A_27a\ 2\ A_27b)).$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \end{aligned} \quad (48)$$

Definition 52 We define `c_Ewords_2Eword_lsl` to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1l \in ty_2Enum_2Enum.$

Definition 53 We define `c_Ewords_2Eword_or` to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1l \in ty_2Enum_2Enum.$

Definition 54 We define `c_Ebool_2ELET` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b.V0f))$

Definition 55 We define `c_2Ewords_2Eword_join` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2)$

Definition 56 We define `c_2Ewords_2Eword_concat` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2)$

Definition 57 We define `c_2Emachine_ieee_2Efloat_to_fp64` to be $\lambda V0x \in (ty_2Eb_2Efloat_2Efloat)$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}). \\ & (\forall V2x \in A.27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A.27c\ A.27b\ A.27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty_2Eb_2Efloat_2Efloat\ (ty_2Efc_2Ebit0\ (\\ & ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit1 \\ & ty_2Eone_2Eone))))))\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0 \\ & ty_2Eone_2Eone))))).((ap\ c_2Emachine_ieee_2Efp64_to_float \\ & (ap\ c_2Emachine_ieee_2Efloat_to_fp64\ V0x)) = V0x)) \end{aligned} \quad (50)$$

Theorem 1

$$\begin{aligned} & ((\forall V0a \in (ty_2Eb_2Efloat_2Efloat\ (ty_2Efc_2Ebit0 \\ & (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit1 \\ & ty_2Eone_2Eone))))))\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0 \\ & ty_2Eone_2Eone))))).((p\ (ap\ c_2Emachine_ieee_2Efp64_isSignallingNan \\ & (ap\ c_2Emachine_ieee_2Efloat_to_fp64\ V0a))) \Leftrightarrow (p\ (ap\ (c_2Eb_2Efloat_is_signalling \\ & (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0 \\ & (ty_2Efc_2Ebit1\ ty_2Eone_2Eone))))))\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit1 \\ & (ty_2Efc_2Ebit0\ ty_2Eone_2Eone))))\ V0a)))) \wedge (\forall V1a \in ty_2Enum_2Enum. \\ & ((p\ (ap\ c_2Emachine_ieee_2Efp64_isSignallingNan\ (ap\ (c_2Ewords_2En2w \\ & (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit0 \\ & (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit0\ ty_2Eone_2Eone))))))\ V1a))) \Leftrightarrow \\ & (p\ (ap\ (c_2Eb_2Efloat_is_signalling\ (ty_2Efc_2Ebit0 \\ & (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit1 \\ & ty_2Eone_2Eone))))))\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit1\ (ty_2Efc_2Ebit0 \\ & ty_2Eone_2Eone))))))\ (ap\ c_2Emachine_ieee_2Efp64_to_float \\ & (ap\ (c_2Ewords_2En2w\ (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit0 \\ & (ty_2Efc_2Ebit0\ (ty_2Efc_2Ebit0\ ty_2Eone_2Eone))))))\ V1a)))))) \end{aligned}$$