

Definition 7 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2EEXP$

Definition 8 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2EEXP$

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 10 We define $c_Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 11 We define $c_Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define c_Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 13 We define c_Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2EfcP_2Efinite_image A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2EfcP_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 14 We define c_Ebool_2EF to be $(ap (c_Ebool_2E.21 2) (\lambda V0t \in 2.V0t))$.

Definition 15 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 18 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A.\lambda y.y \in A)$) of type $\iota \Rightarrow \iota$.

Definition 19 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Definition 20 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (c_2Eprim_rec_2E_3C V0m V1n))$

Definition 21 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C A_27a P)))$

Definition 22 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 A_27a (ty_2Enum_2Enum A_27a)))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (15)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (16)$$

Definition 23 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b)$

Definition 24 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efcp_2EFCP A_27a A_27b g)))$

Definition 25 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcp_2EFCP A_27a A_27a n))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (17)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Ebinary_ieee_2Efloat A0 A1) \quad (18)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign A_27t A_27w \in ((ty_2Efcp_2Ecart 2 ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)}) \quad (19)$$

Let $c_2Ebinary_ieee_2Efloat_Significand_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand_fupd\ A_27t\ A_27u\ A_27w \in \\ & ((ty_2Ebinary_ieee_2Efloat\ A_27u\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (25)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x.nonempty\ A_27x \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A_27t\ A_27w\ A_27x \in \\ & (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (26)$$

Definition 39 We define $c_2Emachine_ieee_2Efp64_to_float$ to be $\lambda V0w \in (ty_2Efc_2Ecart\ 2\ (ty_2Efc_2Ecart\ 2\ A_27a))$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in \\ & ((ty_2Efc_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (27)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent\ A_27t\ A_27w \in \\ & ((ty_2Efc_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (28)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \end{aligned} \quad (29)$$

Definition 40 We define $c_2Ewords_2Eword_lsl$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1x \in (ty_2Efc_2Ecart\ 2\ A_27a)$

Definition 41 We define $c_2Ewords_2Eword_or$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1u \in (ty_2Efc_2Ecart\ 2\ A_27a)$

Definition 42 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b))$

Definition 43 We define $c_2Ewords_2Eword_join$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a)$

Definition 44 We define $c_2Ewords_2Eword_concat$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a)$

Definition 45 We define $c_2Emachine_ieee_2Efloat_to_fp64$ to be $\lambda V0x \in (ty_2Ebinary_ieee_2Efloat\ A_27a)$

Definition 46 We define $c_2Emachine_ieee_2Efp64_negate$ to be $(ap\ (ap\ (c_2Ecombin_2Eo\ (ty_2Efc_2Ecart\ 2\ A_27a))\ (ty_2Efc_2Ecart\ 2\ A_27a))\ (ty_2Efc_2Ecart\ 2\ A_27a))$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\
& \text{nonempty } A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27a^{A_27c}). \\
& (\forall V2x \in A_27c. ((\text{ap } (\text{ap } (\text{ap } (\text{c_2Ecombin_2Eo } A_27c } A_27b } A_27a) \\
& V0f) V1g) V2x) = (\text{ap } V0f (\text{ap } V1g V2x))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (\text{ty_2Ebinary_ieee_2Efloat } (\text{ty_2Efc_2Ebit0 } (\\
& \text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit1 } \\
& \text{ty_2Eone_2Eone})))))) (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } \\
& \text{ty_2Eone_2Eone}))))). ((\text{ap } \text{c_2Emachine_ieee_2Efp64_to_float} \\
& (\text{ap } \text{c_2Emachine_ieee_2Efloat_to_fp64 } V0x)) = V0x))
\end{aligned} \tag{31}$$

Theorem 1

$$\begin{aligned}
& ((\forall V0a \in (\text{ty_2Ebinary_ieee_2Efloat } (\text{ty_2Efc_2Ebit0} \\
& (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit1} \\
& \text{ty_2Eone_2Eone})))))) (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0} \\
& \text{ty_2Eone_2Eone}))))). ((\text{ap } \text{c_2Emachine_ieee_2Efp64_negate} \\
& (\text{ap } \text{c_2Emachine_ieee_2Efloat_to_fp64 } V0a)) = (\text{ap } \text{c_2Emachine_ieee_2Efloat_to_fp64} \\
& (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_negate } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0} \\
& (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit1 } \text{ty_2Eone_2Eone})))))) \\
& (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone})))) \\
& V0a)))) \wedge (\forall V1a \in \text{ty_2Enum_2Enum}. ((\text{ap } \text{c_2Emachine_ieee_2Efp64_negate} \\
& (\text{ap } (\text{c_2Ewords_2En2w } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0} \\
& (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone})))))) \\
& V1a)) = (\text{ap } \text{c_2Emachine_ieee_2Efloat_to_fp64} (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_negate} \\
& (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit0} \\
& (\text{ty_2Efc_2Ebit1 } \text{ty_2Eone_2Eone})))))) (\text{ty_2Efc_2Ebit1 } (\text{ty_2Efc_2Ebit1} \\
& (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone})))) (\text{ap } \text{c_2Emachine_ieee_2Efp64_to_float} \\
& (\text{ap } (\text{c_2Ewords_2En2w } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0} \\
& (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } (\text{ty_2Efc_2Ebit0 } \text{ty_2Eone_2Eone})))))) \\
& V1a))))))
\end{aligned}$$