

thm_2Emachine__ieee_2Efp64__sqrt (TMXS- MGikrXMkETQXk32ZYbUNWB7S8NpCt97)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E$

Definition 8 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 10 We define $c_Ebit_EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_Earithmetic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 11 We define $c_Ebit_EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define c_Ebit_EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 13 We define c_Ebit_EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efc_2Efinite_image A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Efc_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 14 We define c_Ebool_2EF to be $(ap (c_Ebool_2E.21 2)) (\lambda V0t \in 2.V0t)$.

Definition 15 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.(ap (c_2Emin_2E_3D_3D_3E V2t) c_2Ebool_2E_7E))))$

Definition 18 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A.\lambda y.y \in A))$ of type $\iota \Rightarrow \iota$.

Definition 19 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) P)))$

Definition 20 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (c_2Emin_2E_40 A_27a) m)$

Definition 21 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C A_27a) P))$

Definition 22 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 A_27a) (ty_2Enum_2Enum A_27a))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (15)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (16)$$

Definition 23 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b)$

Definition 24 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efcp_2Efcp_index A_27a A_27b) g))$

Definition 25 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcp_2EFCP A_27a A_27a) n)$

Let $ty_2Ebinary_ieee_2Efp_op : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Ebinary_ieee_2Efp_op A0 A1) \quad (17)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Ebinary_ieee_2Efloat A0 A1) \quad (18)$$

Let $ty_2Ebinary_ieee_2Errounding : \iota$ be given. Assume the following.

$$nonempty ty_2Ebinary_ieee_2Errounding \quad (19)$$

Let $c_2Ebinary_ieee_2EFP_Sqrt : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2EFP_Sqrt\ A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})_{ty_2Ebin}) \quad (20)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (21)$$

Definition 26 We define $c_2Ewords_2Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (22)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \quad (23)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealx_2Ereal^{ty_2Enum_2Enum})_{ty_2Erealx_2Ereal}) \quad (24)$$

Definition 27 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Definition 28 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})_{ty_2Enum_2Enum}) \quad (25)$$

Definition 29 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap\ (ap\ c$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (26)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (27)$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealx_2Ereal}) \quad (28)$$

Definition 30 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E.40 (t$
Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_inv \in & ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \end{aligned} \quad (29)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) \quad (30)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}} \quad (31)$$

Definition 31 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 32 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_mul \in & (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal) \end{aligned} \quad (32)$$

Definition 33 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 34 We define $c_2Ereal_2E.2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_add \in & (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal) \end{aligned} \quad (33)$$

Definition 35 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2EINT_MAX A_27a \in (ty_2Eenum_2Eenum)^{(ty_2Ebool_2Eitself A_27a)} \quad (34)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent \\ A_27t A_27w \in ((ty_2EfcP_2Ecart 2 A_27w)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)}) \end{aligned} \quad (35)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (36)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (37)$$

Let $c_2Erealax_2Etreax_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (38)$$

Definition 36 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Definition 37 We define $c_2Ebinary_ieee_2Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary$

Let $ty_2Ebinary_ieee_2Efloat_value : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_value \quad (39)$$

Let $c_2Ebinary_ieee_2Efloat : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Efloat \in (ty_2Ebinary_ieee_2Efloat_value^{ty_2Erealax_2Ereal}) \quad (40)$$

Let $c_2Ebinary_ieee_2ENaN : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2ENaN \in ty_2Ebinary_ieee_2Efloat_value \quad (41)$$

Let $c_2Ebinary_ieee_2EInfinity : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EInfinity \in ty_2Ebinary_ieee_2Efloat_value \quad (42)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Eenum_2Eenum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (43)$$

Definition 38 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap\ (c_2Ewords_2Een2w\ A_27a)\ (ap\ (c_2Ew$

Definition 39 We define $c_2Ebinary_ieee_2Efloat_value$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary$

Let $c_2Ebinary_ieee_2Efloat_value_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebinary_ieee_2Efloat_value_CASE\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebinary_ieee_2Efloat_value} \quad (44)$$

Definition 40 We define $c_2Ebinary_ieee_2Efloat_is_nan$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary$

Definition 41 We define $c_2Ebinary_ieee_2Efloat_is_signalling$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary$

Definition 42 We define `c2Ebool_2ELET` to be $\lambda A.27a : \iota. \lambda A.27b : \iota. (\lambda V0f \in (A.27b^{A.27a}). (\lambda V1x \in A.27b. V0f x))$

Definition 43 We define `c2Ebinary_2Efloat_2Esome_2Eqlnan` to be $\lambda A.27t : \iota. \lambda A.27w : \iota. \lambda V0fp_op \in (ty \rightarrow \iota. \iota)$

Let `ty_2Ebinary_2Eflags` : ι be given. Assume the following.

$$\text{nonempty } ty_2Ebinary_2Eflags \quad (45)$$

Let `c_2Ebool_2EARB` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a. \text{nonempty } A.27a \Rightarrow c_2Ebool_2EARB \ A.27a \in A.27a \quad (46)$$

Definition 44 We define `c_2Ecombin_2EK` to be $\lambda A.27a : \iota. \lambda A.27b : \iota. (\lambda V0x \in A.27a. (\lambda V1y \in A.27b. V0x y))$

Let `c_2Ebinary_2Eflags_2EUnderflow_2EAfterRounding_2Efuld` : ι be given. Assume the following.

$$c_2Ebinary_2Eflags_2EUnderflow_2EAfterRounding_2Efuld \in ((ty_2Ebinary_2Eflags^{ty_2Ebinary_2Eflags})^{(2^2)}) \quad (47)$$

Let `c_2Ebinary_2Eflags_2EUnderflow_2EBeforeRounding_2Efuld` : ι be given. Assume the following.

$$c_2Ebinary_2Eflags_2EUnderflow_2EBeforeRounding_2Efuld \in ((ty_2Ebinary_2Eflags^{ty_2Ebinary_2Eflags})^{(2^2)}) \quad (48)$$

Let `c_2Ebinary_2Eflags_2EPrecision_2Efuld` : ι be given. Assume the following.

$$c_2Ebinary_2Eflags_2EPrecision_2Efuld \in ((ty_2Ebinary_2Eflags^{ty_2Ebinary_2Eflags})^{(2^2)}) \quad (49)$$

Let `c_2Ebinary_2Eflags_2EOverflow_2Efuld` : ι be given. Assume the following.

$$c_2Ebinary_2Eflags_2EOverflow_2Efuld \in ((ty_2Ebinary_2Eflags^{ty_2Ebinary_2Eflags})^{(2^2)}) \quad (50)$$

Let `c_2Ebinary_2Eflags_2EInvalidOp_2Efuld` : ι be given. Assume the following.

$$c_2Ebinary_2Eflags_2EInvalidOp_2Efuld \in ((ty_2Ebinary_2Eflags^{ty_2Ebinary_2Eflags})^{(2^2)}) \quad (51)$$

Let `c_2Ebinary_2Eflags_2EDivideByZero_2Efuld` : ι be given. Assume the following.

$$c_2Ebinary_2Eflags_2EDivideByZero_2Efuld \in ((ty_2Ebinary_2Eflags^{ty_2Ebinary_2Eflags})^{(2^2)}) \quad (52)$$

Definition 45 We define `c_2Ebinary_2Eclear_2Eflags` to be $(\text{ap } (\text{ap } c_2Ebinary_2Eflags_2EDivideByZero_2Efuld))$

Definition 46 We define `c_2Ebinary_2Einvalidop_2Eflags` to be $(\text{ap } (\text{ap } c_2Ebinary_2Eflags_2EInvalidOp_2Efuld))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}})$$
(53)

Definition 47 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ x\ y)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0)$$
(54)

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a)$$
(55)

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a})$$
(56)

Let $c_2Elist_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEXISTS\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})})$$
(57)

Definition 48 We define $c_2Ebinary_ieee_2Echeck_for_signalling$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0l \in (ty_2Ebinary_ieee_2Efloat_plus_infinity\ A_27a\ A_27b)$

Let $c_2Ebinary_ieee_2Efloat_plus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_infinity\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))})$$
(58)

Let $c_2Erealx_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
(59)

Definition 49 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$

Definition 50 We define $c_2Etransc_2Eroot$ to be $\lambda V0n \in ty_2Enum_2Enum.\lambda V1x \in ty_2Erealx_2Ereal$

Definition 51 We define $c_2Etransc_2Esqrt$ to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap\ (ap\ c_2Etransc_2Eroot\ x))$

Definition 52 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$

Definition 53 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONJ\ x\ 0))))$

Let $c_2Ebinary_ieee_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Elargest \\ & A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (60)$$

Definition 54 We define $c_2Ebinary_ieee_2Efloat_is_finite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_is_finite\ A_27t\ A_27w)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (61)$$

Definition 55 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 56 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 57 We define $c_2Ebinary_ieee_2Eis_closest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{(ty_2Ebinary_ieee_2Efloat_is_finite\ A_27a\ A_27b)})$

Definition 58 We define $c_2Ebinary_ieee_2Eclosest_such$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_2Ebinary_ieee_2Efloat_is_finite\ A_27a\ A_27b)})$

Definition 59 We define $c_2Ebinary_ieee_2Eclosest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Ebinary_ieee_2Eclosest_such\ A_27a\ A_27b))$

Let $c_2Ebinary_ieee_2Efloat_top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_top \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (62)$$

Definition 60 We define $c_2Ereal_2Ereal_gt$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ebinary_ieee_2Efloat_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_bottom \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (63)$$

Let $c_2Ebinary_ieee_2Efloat_minus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_infinity \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (64)$$

Definition 61 We define $c_2Ereal_2Ereal_ge$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ebinary_ieee_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Ethreshold \\ & A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (65)$$

Definition 62 We define $c_Ewords_Eword_lsb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap$
 Let $c_2Ebinary_ieee_2Erounding2num : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Erounding2num \in (ty_2Enum_2Enum^{ty_2Ebinary_ieee_2Erounding}) \quad (66)$$

Definition 63 We define $c_2Ebinary_ieee_2Erounding_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_2Ebinary_ieee_2E$

Definition 64 We define $c_2Ebinary_ieee_2ERound$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinary_ie$

Let $c_2Ebinary_ieee_2Efloat_plus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_zero\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)})) \quad (67)$$

Let $c_2Ebinary_ieee_2Efloat_minus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_zero\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)})) \quad (68)$$

Definition 65 We define $c_2Ebinary_ieee_2Efloat_is_zero$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Definition 66 We define $c_2Ebinary_ieee_2Efloat_round$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebin$

Let $c_2Ewords_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MIN\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (69)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (70)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (71)$$

Definition 67 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Definition 68 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 69 We define $c_2Ewords_2Enzcv$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1b \in ($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (72)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (73)$$

Definition 70 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 71 We define $c_2Ewords_2Eword_ls$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1b$

Definition 72 We define $c_2Ebinary_ieee_2Efloat_is_infinite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Definition 73 We define $c_2Ebinary_ieee_2Efloat_round_with_flags$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode$

Definition 74 We define $c_2Ebinary_ieee_2Efloat_sqrt$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinar$

Definition 75 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $ty_2EfcP_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit0\ A0) \quad (74)$$

Let $ty_2EfcP_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit1\ A0) \quad (75)$$

Definition 76 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2En$

Definition 77 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Definition 78 We define $c_2Ewords_2Eword_bits$ to be $\lambda A_27a : \iota.\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2$

Definition 79 We define $c_2Ewords_2Ew2w$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a$

Definition 80 We define $c_2Ewords_2Eword_extract$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0h \in ty_2Enum_2Enum$

Let $c_2Ebinary_ieee_2Efloat_Significand_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27v \\ nonempty\ A_27v \Rightarrow c_2Ebinary_ieee_2Efloat_Significand_f \\ A_27t\ A_27u\ A_27v \in (((ty_2Ebina$$

(76)

Let $c_2Ebinary_ieee_2Efloat_Exponent_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x. \\ & \quad nonempty\ A_27x \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A_27t\ A_27w\ A_27x \in \\ & \quad ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{((ty_2Efloat\ A_27t\ A_27x)^{(ty_2Efloat\ A_27t\ A_27w)})} \\ & \quad (77) \end{aligned}$$

Let $c_2Ebinary_ieee_2Efloat_Sign_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign_fupd\ A_27t\ A_27w \in \\ & \quad ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{((ty_2Efloat\ A_27t\ A_27w)^{(ty_2Efloat\ A_27t\ A_27w)})} \\ & \quad (78) \end{aligned}$$

Definition 81 We define $c_2Emachine_ieee_2Efp64_to_float$ to be $\lambda V0w \in (ty_2Efloat_2Ecart\ 2\ (ty_2Efloat\ 2\ A_27a))$.

Definition 82 We define $c_2Ewords_2Eword_lsl$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efloat_2Ecart\ 2\ A_27a).\lambda V1v \in (ty_2Efloat\ 2\ A_27a)$.

Definition 83 We define $c_2Ewords_2Eword_or$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efloat_2Ecart\ 2\ A_27a).\lambda V1v \in (ty_2Efloat\ 2\ A_27a)$.

Definition 84 We define $c_2Ewords_2Eword_join$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0v \in (ty_2Efloat_2Ecart\ 2\ A_27a)$.

Definition 85 We define $c_2Ewords_2Eword_concat$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0v \in (ty_2Efloat_2Ecart\ 2\ A_27a)$.

Definition 86 We define $c_2Emachine_ieee_2Efloat_to_fp64$ to be $\lambda V0x \in (ty_2Ebinary_ieee_2Efloat\ (ty_2Efloat\ 2\ A_27a))$.

Definition 87 We define $c_2Emachine_ieee_2Efp64_sqrt$ to be $\lambda V0mode \in ty_2Ebinary_ieee_2ERounding$.

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\ & \quad (\forall V2x \in A_27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a))\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \\ & \quad (79) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty_2Ebinary_ieee_2Efloat\ (ty_2Efloat_2Ebit0\ (\\ & \quad ty_2Efloat_2Ebit0\ (ty_2Efloat_2Ebit1\ (ty_2Efloat_2Ebit0\ (ty_2Efloat_2Ebit1 \\ & \quad ty_2Eone_2Eone))))))\ (ty_2Efloat_2Ebit1\ (ty_2Efloat_2Ebit1\ (ty_2Efloat_2Ebit0 \\ & \quad ty_2Eone_2Eone))))).((ap\ c_2Emachine_ieee_2Efp64_to_float \\ & \quad (ap\ c_2Emachine_ieee_2Efloat_to_fp64\ V0x)) = V0x) \\ & \quad (80) \end{aligned}$$

Theorem 1

$$\begin{aligned}
& ((\forall V0mode \in ty_2Ebinary_ieee_2Errounding. (\forall V1a \in \\
& \quad (ty_2Ebinary_ieee_2Efloat (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& \quad (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 ty_2Eone_2Eone)))) \\
& \quad (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone))))). \\
& ((ap (ap c_2Emachine_ieee_2Efp64_sqrt V0mode) (ap c_2Emachine_ieee_2Efloat_to_fp64 \\
& \quad V1a)) = (ap c_2Emachine_ieee_2Efloat_to_fp64 (ap (c_2Epair_2ESND \\
& \quad ty_2Ebinary_ieee_2Eflags (ty_2Ebinary_ieee_2Efloat (ty_2Efc_2Ebit0 \\
& \quad (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 \\
& \quad ty_2Eone_2Eone)))))) (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 \\
& \quad ty_2Eone_2Eone)))))) (ap (ap (c_2Ebinary_ieee_2Efloat_sqrt \\
& \quad (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 \\
& \quad (ty_2Efc_2Ebit1 ty_2Eone_2Eone)))))) (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 \\
& \quad (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) V0mode) V1a)))))) \wedge (\forall V2mode \in \\
& \quad ty_2Ebinary_ieee_2Errounding. (\forall V3a \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Emachine_ieee_2Efp64_sqrt V2mode) (ap (c_2Ewords_2En2w \\
& \quad (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& \quad (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) V3a)) = \\
& \quad (ap c_2Emachine_ieee_2Efloat_to_fp64 (ap (c_2Epair_2ESND \\
& \quad ty_2Ebinary_ieee_2Eflags (ty_2Ebinary_ieee_2Efloat (ty_2Efc_2Ebit0 \\
& \quad (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 \\
& \quad ty_2Eone_2Eone)))))) (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 \\
& \quad ty_2Eone_2Eone)))))) (ap (ap (c_2Ebinary_ieee_2Efloat_sqrt \\
& \quad (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 \\
& \quad (ty_2Efc_2Ebit1 ty_2Eone_2Eone)))))) (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 \\
& \quad (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) V2mode) (ap c_2Emachine_ieee_2Efp64_to_float \\
& \quad (ap (c_2Ewords_2En2w (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\
& \quad (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) V3a)))))))))
\end{aligned}$$