

thm_2Emachine_ieee_2Efp64_to_int
 (TMQCMlswckrmYGx-
 oyos8qhQrrvsULtDBdKA)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 10 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 11 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 13 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efc_2Efinite_image A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efc_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 14 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 15 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 16 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_21\ 2))$

Definition 17 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Definition 18 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 19 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40 A_27a))))$

Definition 20 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_Ebool_2E_2F_5C A_27a P)))$

Definition 22 We define $c_Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40 (A_27a^{ty_2Enum_2Enum})))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcp_2Ecart\ A0\ A1) \quad (15)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcp_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \quad (16)$$

Definition 23 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Definition 24 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efcp_2Efcp_index\ A_27a\ A_27b\ g)))$

Definition 25 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcp_2EFCP\ A_27a\ n))$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (17)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (18)$$

Definition 26 We define c_2Eone_2Eone to be $(ap (c_Emin_2E_40\ ty_2Eone_2Eone)) (\lambda V0x \in ty_2Eone_2Eone)$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (19)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (20)$$

Definition 27 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (21)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (22)$$

Definition 28 We define $c_2Eoption_2Eoption_ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ 0)$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (23)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (24)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (25)$$

Definition 29 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint. (ap\ (c_2Emin_2E40\ ty_2Einteger_2Eint_REP_CLASS)\ V0a)$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (26)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (27)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \quad (28)$$

Definition 30 We define $c_Einteger_Eint_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_E$

Definition 31 We define $c_Einteger_Eint_add$ to be $\lambda V0T1 \in ty_Einteger_Eint.\lambda V1T2 \in ty_Eintege$

Definition 32 We define $c_Einteger_EEnum$ to be $\lambda V0i \in ty_Einteger_Eint.(ap\ (c_Emin_E_40\ ty_E$

Let $ty_Erealax_Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_Erealax_Ereal \quad (29)$$

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Eenum_Eenum}) \quad (30)$$

Let $c_Einteger_Eint_neg : \iota$ be given. Assume the following.

$$c_Einteger_Eint_neg \in ((ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)}) \quad (31)$$

Definition 33 We define $c_Einteger_Eint_neg$ to be $\lambda V0T1 \in ty_Einteger_Eint.(ap\ c_Einteger_Eint$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \quad (32)$$

Let $c_Erealax_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Erealax_Ereal}) \quad (33)$$

Definition 34 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal.(ap\ (c_Emin_E_40\ ($

Let $c_Erealax_Ereal_neg : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_neg \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (34)$$

Let $c_Erealax_Ereal_eq : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_eq \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (35)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal^{(2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})}) \quad (36)$$

Definition 35 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_E$

Definition 36 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum)})$$
(37)

Definition 37 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$.

Definition 38 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 39 We define $c_2Eintreal_2Ereal_of_int$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (c_2Eboo$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal)})$$
(38)

Definition 40 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$.

Definition 41 We define $c_2Einteger_2ELEAST_INT$ to be $\lambda V0P \in (2^{ty_2Einteger_2Eint}).(ap\ (c_2Emin_2E4$

Definition 42 We define $c_2Eintreal_2EINT_FLOOR$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ c_2Einteger_2E$

Definition 43 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Er$

Definition 44 We define $c_2Eintreal_2EINT_CEILING$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ c_2Einteger_2E$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1)$$
(39)

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign\ A_27t\ A_27w \in ((ty_2Efcpl_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})$$
(40)

Let $c_2Erealax_2Etrealt_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
(41)

Definition 45 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 46 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Er$

Definition 47 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Eboo$

Definition 48 We define $c_2Einteger_2EABS$ to be $\lambda V0n \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (c_2Eboo$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (42)$$

Let $c_2Erealax_2Ereal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (43)$$

Definition 49 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Ereal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (44)$$

Definition 50 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 51 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 52 We define $c_2Ebool_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Definition 53 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b.$

Let $ty_2Ebinary_ieee_2Erounding : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Erounding \quad (45)$$

Let $c_2Ebinary_ieee_2Erounding2num : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Erounding2num \in (ty_2Enum_2Enum^{ty_2Ebinary_ieee_2Erounding}) \quad (46)$$

Definition 54 We define $c_2Ebinary_ieee_2Erounding_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_2Ebinary_ieee_2Erounding2num.$

Definition 55 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Definition 56 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_2EABS$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (47)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.\ nonempty\ A_27t \Rightarrow \forall A_27w.\ nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (48)$$

Definition 57 We define `c_2Ebit_2ESBIT` to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebo$
Let `c_2Esum_num_2ESUM` : ι be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (49)$$

Definition 58 We define `c_2Ewords_2Ew2n` to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap (ap$
Let `c_2Ewords_2EINT_MAX` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (50)$$

Let `c_2Ebinary_ieee_2Efloat_Exponent` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (51)$$

Definition 59 We define `c_2Ebinary_ieee_2Efloat_to_real` to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$
Let `ty_2Ebinary_ieee_2Efloat_value` : ι be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_value \quad (52)$$

Let `c_2Ebinary_ieee_2EFloat` : ι be given. Assume the following.

$$c_2Ebinary_ieee_2EFloat \in (ty_2Ebinary_ieee_2Efloat_value^{ty_2Erealax_2Ereal}) \quad (53)$$

Let `c_2Ebinary_ieee_2ENaN` : ι be given. Assume the following.

$$c_2Ebinary_ieee_2ENaN \in ty_2Ebinary_ieee_2Efloat_value \quad (54)$$

Let `c_2Ebinary_ieee_2EInfinity` : ι be given. Assume the following.

$$c_2Ebinary_ieee_2EInfinity \in ty_2Ebinary_ieee_2Efloat_value \quad (55)$$

Let `c_2Ewords_2EUINT_MAX` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (56)$$

Definition 60 We define `c_2Ewords_2Eword_T` to be $\lambda A_27a : \iota.(ap (c_2Ewords_2En2w\ A_27a) (ap (c_2Ew$

Definition 61 We define `c_2Ebinary_ieee_2Efloat_value` to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Let `c_2Ebinary_ieee_2Efloat_value_CASE` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebinary_ieee_2Efloat_value_CASE\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebinary_ieee_2Efloat_value} \quad (57)$$

Definition 71 We define $c_Emachine_ieee_Efp64_to_int$ to be $\lambda V0mode \in ty_Ebinary_ieee_ERounding$

Definition 72 We define $c_Ewords_Eword_lsl$ to be $\lambda A.27a : \iota.\lambda V0w \in (ty_Efcpcart2\ A.27a).\lambda V1w$

Definition 73 We define $c_Ewords_Eword_or$ to be $\lambda A.27a : \iota.\lambda V0v \in (ty_Efcpcart2\ A.27a).\lambda V1v$

Definition 74 We define $c_Ewords_Eword_join$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0v \in (ty_Efcpcart2\ A.27a)$

Definition 75 We define $c_Ewords_Eword_concat$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0v \in (ty_Efcpcart2\ A.27a)$

Definition 76 We define $c_Emachine_ieee_Efloat_to_fp64$ to be $\lambda V0x \in (ty_Ebinary_ieee_Efloat\ (ty_Eone_Eone))$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}). \\ & (\forall V2x \in A.27c.((ap\ (ap\ (ap\ (c_Ecombin_Eo\ A.27c\ A.27b\ A.27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty_Ebinary_ieee_Efloat\ (ty_Efcpcart2\ Ebit0\ (\\ & ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit1 \\ & ty_Eone_Eone))))))\ (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit0 \\ & ty_Eone_Eone))))).((ap\ c_Emachine_ieee_Efp64_to_float \\ & (ap\ c_Emachine_ieee_Efloat_to_fp64\ V0x)) = V0x)) \end{aligned} \quad (65)$$

Theorem 1

$$\begin{aligned} & ((\forall V0mode \in ty_Ebinary_ieee_ERounding.(\forall V1a \in \\ & (ty_Ebinary_ieee_Efloat\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit0 \\ & (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit1\ ty_Eone_Eone)))))) \\ & (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit0\ ty_Eone_Eone))))). \\ & ((ap\ (ap\ c_Emachine_ieee_Efp64_to_int\ V0mode)\ (ap\ c_Emachine_ieee_Efloat_to_fp64 \\ & V1a)) = (ap\ (ap\ (c_Ebinary_ieee_Efloat_to_int\ (ty_Efcpcart2\ Ebit0 \\ & (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit1 \\ & ty_Eone_Eone))))))\ (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit0 \\ & ty_Eone_Eone))))))\ V0mode)\ V1a)))) \wedge (\forall V2mode \in ty_Ebinary_ieee_ERounding. \\ & (\forall V3a \in ty_Eenum_Eenum.((ap\ (ap\ c_Emachine_ieee_Efp64_to_int \\ & V2mode)\ (ap\ (c_Ewords_Een2w\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit0 \\ & (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit0 \\ & ty_Eone_Eone))))))\ V3a)) = (ap\ (ap\ (c_Ebinary_ieee_Efloat_to_int \\ & (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit0 \\ & (ty_Efcpcart2\ Ebit1\ ty_Eone_Eone))))))\ (ty_Efcpcart2\ Ebit1\ (ty_Efcpcart2\ Ebit1 \\ & (ty_Efcpcart2\ Ebit0\ ty_Eone_Eone))))))\ V2mode)\ (ap\ c_Emachine_ieee_Efp64_to_float \\ & (ap\ (c_Ewords_Een2w\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit0 \\ & (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit0\ (ty_Efcpcart2\ Ebit0\ ty_Eone_Eone))))))\ \\ & V3a)))))) \end{aligned}$$