

# thm\_2Emeasure\_2EADDITIVE\_SUM (TMa7Gq9r7JswxnJNGVLR2MxQn15c28hzVfN)

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Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E21$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A\_27a}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))))$

**Definition 4** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in (A\_27c^{A\_27b}).(c\_2Emin\_2E3D\ (A\_27a\ (A\_27b\ A\_27c)))$

**Definition 5** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 6** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 7** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a}))\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27a)))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{4}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}\ ty\_2Erealax\_2Ereal)) \tag{5}$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 9** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty$   
Let  $c\_2Erealax\_2Ereal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)) \quad (6)$$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

**Definition 11** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 12** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21))$ .

**Definition 14** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$ .

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Emeasure A\_27a \in ( (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))) \quad (7)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (8)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (10)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets A\_27a \in ((2^{(2^{A\_27a})})(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \quad (12)$$



Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (18)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (19)$$

**Definition 27** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 29** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EINTER))$

**Definition 30** We define  $c\_2Epred\_set\_2EEDISJOINT$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EEDISJOINT))$

**Definition 31** We define  $c\_2Emeasure\_2Eadditive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a}))\ (ty\_2Epair\_2Eprod\ (2^{A\_27a}))$

**Definition 32** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E2ET)$ .

**Definition 33** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E40))))$

**Definition 34** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A\_27b}).(ap\ (c\_2Epred\_set\_2EFUNSET))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (20)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (21)$$

**Definition 35** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num)$

**Definition 36** We define  $c\_2Eprim\_rec\_2E3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 37** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EINSERT))$

**Definition 38** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E21\ 2))$

**Definition 39** We define  $c\_2Epred\_set\_2Ecount$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Epred\_set\_2EG))$

**Definition 40** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27b}).(ap\ (c\_2Epred\_set\_2EIMAGE))$

**Definition 41** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_set\_2EBIGUNION))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (22)$$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ & ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\ & (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\ & V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ & V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))) \end{aligned} \quad (24)$$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t) \Leftrightarrow (p V0t))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\ & ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \end{aligned} \quad (30)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (35)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Rightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (40)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\neg(\forall V1x \in A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (41)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p \ V0P) \wedge (\forall V2x \in A\_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x))))))) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x)))))) \quad (43)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q)))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C)))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (48)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (49)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (50)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}). (\forall V4x \in A.27a. (p (ap (ap V0P V4x) (ap V3f V4x)))))) \quad (51)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}). \\
& (\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27c\ A\_27b\ A\_27a) \\
& V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Ecombin\_2EI \\
A\_27a)\ V0x) = V0x)) \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in \\
& (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E2C \\
& (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V1y)))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a})\ (2^{(2^{A\_27a})})).(\forall V1c \in (2^{(2^{A\_27a})}).(((p\ ( \\
& ap\ (c\_2Emeasure\_2Ealgebra\ A\_27a)\ V0a)) \wedge ((p\ (ap\ (c\_2Epred\_set\_2EFINITE \\
& (2^{A\_27a})\ V1c)) \wedge (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ (2^{A\_27a}) \\
& V1c)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& (2^{A\_27a})\ (ap\ (c\_2Epred\_set\_2EBIGUNION\ A\_27a)\ V1c))\ (ap\ (c\_2Emeasure\_2Esubsets \\
& A\_27a)\ V0a))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). \\
& (\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).(\forall V3u \in \\
& (2^{A\_27a}).(((p\ (ap\ (c\_2Emeasure\_2Eadditive\ A\_27a)\ V0m)) \wedge ((p \\
& (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a})\ V1s)\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets \\
& A\_27a)\ V0m))) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a})\ V2t)\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets \\
& A\_27a)\ V0m))) \wedge ((p\ (ap\ (ap\ (c\_2Epred\_set\_2EDISJOINT\ A\_27a)\ V1s) \\
& V2t)) \wedge (V3u = (ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a)\ V1s)\ V2t)))))) \Rightarrow \\
& ((ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m)\ V3u) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\
& (ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m)\ V1s))\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure \\
& A\_27a)\ V0m)\ V2t))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))))
\end{aligned} \tag{57}$$



Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (p (ap (ap (c.2Ebool.2EIN A.27a) V0x) (c.2Epred\_set.2EUNIV A.27a)))) \quad (58)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & \quad \forall V0y \in A.27b. (\forall V1s \in (2^{A.27a}). (\forall V2f \in (A.27b^{A.27a}). \\ & \quad ((p (ap (ap (c.2Ebool.2EIN A.27b) V0y) (ap (ap (c.2Epred\_set.2EIMAGE \\ & \quad A.27a A.27b) V2f) V1s))) \Leftrightarrow (\exists V3x \in A.27a. ((V0y = (ap V2f V3x)) \wedge \\ & \quad (p (ap (ap (c.2Ebool.2EIN A.27a) V3x) V1s))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & \quad \forall V0f \in (A.27b^{A.27a}). ((ap (ap (c.2Epred\_set.2EIMAGE A.27a \\ & \quad A.27b) V0f) (c.2Epred\_set.2EEMPTY A.27a)) = (c.2Epred\_set.2EEMPTY \\ & \quad A.27b))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & \quad \forall V0f \in (A.27b^{A.27a}). (\forall V1x \in A.27a. (\forall V2s \in ( \\ & \quad 2^{A.27a}). ((ap (ap (c.2Epred\_set.2EIMAGE A.27a A.27b) V0f) (ap \\ & \quad (ap (c.2Epred\_set.2EINSERT A.27a) V1x) V2s)) = (ap (ap (c.2Epred\_set.2EINSERT \\ & \quad A.27b) (ap V0f V1x)) (ap (ap (c.2Epred\_set.2EIMAGE A.27a A.27b) \\ & \quad V0f) V2s)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & \quad \forall V0f \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27a}). (\forall V2Q \in \\ & \quad (2^{A.27b}). ((p (ap (ap (c.2Ebool.2EIN (A.27b^{A.27a}) V0f) (ap (ap \\ & \quad (c.2Epred\_set.2EFUNSET A.27a A.27b) V1P) V2Q))) \Leftrightarrow (\forall V3x \in \\ & \quad A.27a. ((p (ap (ap (c.2Ebool.2EIN A.27a) V3x) V1P)) \Rightarrow (p (ap (ap (c.2Ebool.2EIN \\ & \quad A.27b) (ap V0f V3x)) V2Q)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & \quad \forall V0s \in (2^{A.27a}). ((p (ap (c.2Epred\_set.2EFINITE A.27a) \\ & \quad V0s)) \Rightarrow (\forall V1f \in (A.27b^{A.27a}). (p (ap (c.2Epred\_set.2EFINITE \\ & \quad A.27b) (ap (ap (c.2Epred\_set.2EIMAGE A.27a A.27b) V1f) V0s)))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty.2Enum.2Enum. (\forall V1n \in ty.2Enum.2Enum. ( \\ & \quad (p (ap (ap (c.2Ebool.2EIN ty.2Enum.2Enum) V0m) (ap c.2Epred\_set.2Ecount \\ & \quad V1n))) \Leftrightarrow (p (ap (ap c.2Eprim\_rec.2E.3C V0m) V1n)))) \end{aligned} \quad (64)$$

Assume the following.

$$((ap\ c\_2Epred\_set\_2Ecount\ c\_2Enum\_2E0) = (c\_2Epred\_set\_2EEMPTY\ ty\_2Enum\_2Enum)) \quad (65)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap\ c\_2Epred\_set\_2Ecount\ (ap\ c\_2Enum\_2ESUC\ V0n)) = (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ ty\_2Enum\_2Enum)\ V0n)\ (ap\ c\_2Epred\_set\_2Ecount\ V0n)))) \quad (66)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p\ (ap\ (c\_2Epred\_set\_2EFINITE\ ty\_2Enum\_2Enum)\ (ap\ c\_2Epred\_set\_2Ecount\ V0n)))) \quad (67)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Epred\_set\_2EBIGUNION\ A\_27a)\ (c\_2Epred\_set\_2EEMPTY\ (2^{A\_27a}))) = (c\_2Epred\_set\_2EEMPTY\ A\_27a)) \quad (68)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1P \in (2^{(2^{A\_27a})}). ((ap\ (c\_2Epred\_set\_2EBIGUNION\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ (2^{A\_27a})\ V0s)\ V1P)) = (ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a)\ V0s)\ (ap\ (c\_2Epred\_set\_2EBIGUNION\ A\_27a)\ V1P)))))) \quad (69)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((2^{A\_27a})ty\_2Enum\_2Enum). ((\forall V1m \in ty\_2Enum\_2Enum. (\forall V2n \in ty\_2Enum\_2Enum. ((\neg(V1m = V2n)) \Rightarrow (p\ (ap\ (ap\ (c\_2Epred\_set\_2EDISJOINT\ A\_27a)\ (ap\ V0f\ V1m))\ (ap\ V0f\ V2n)))))) \Rightarrow (\forall V3n \in ty\_2Enum\_2Enum. (p\ (ap\ (ap\ (c\_2Epred\_set\_2EDISJOINT\ A\_27a)\ (ap\ V0f\ V3n))\ (ap\ (c\_2Epred\_set\_2EBIGUNION\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ ty\_2Enum\_2Enum\ (2^{A\_27a})\ V0f)\ (ap\ c\_2Epred\_set\_2Ecount\ V3n)))))))))) \quad (70)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. ((ap\ (ap\ c\_2Erealx\_2Ereal\_add\ V0x)\ V1y) = (ap\ (ap\ c\_2Erealx\_2Ereal\_add\ V1y)\ V0x)))) \quad (71)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum. (\forall V1f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& ((ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum) V0n) c\_2Enum\_2E0)) V1f) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (\forall V3m \in \\
& ty\_2Enum\_2Enum. (\forall V4f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& ((ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum) V2n) (ap c\_2Enum\_2ESUC V3m))) V4f) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum) V2n) V3m))) V4f)) (ap V4f (ap (ap c\_2Earithmic\_2E\_2B \\
& V2n) V3m))))))))))
\end{aligned} \tag{72}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{73}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{76}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge ( \\
& \neg(p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{82}$$

**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealx\_2Ereal(2^{A.27a}))))). \\
& (\forall V1f \in ((2^{A.27a})^{ty\_2Enum\_2Enum}). (\forall V2n \in ty\_2Enum\_2Enum. \\
& (((p (ap (c.2Emeasure\_2Ealgebra \ A.27a) (ap (ap (c.2Epair\_2E\_2C \\
& (2^{A.27a}) (2^{(2^{A.27a})})) (ap (c.2Emeasure\_2Em\_space \ A.27a) \\
& V0m)) (ap (c.2Emeasure\_2Emeasurable\_sets \ A.27a) V0m)))) \wedge (( \\
& p (ap (c.2Emeasure\_2Epositive \ A.27a) V0m)) \wedge ((p (ap (c.2Emeasure\_2Eadditive \\
& A.27a) V0m)) \wedge ((p (ap (ap (c.2Ebool\_2EIN ((2^{A.27a})^{ty\_2Enum\_2Enum})) \\
& V1f) (ap (ap (c.2Epred\_set\_2EFUNSET \ ty\_2Enum\_2Enum (2^{A.27a})) \\
& (c.2Epred\_set\_2EUNIV \ ty\_2Enum\_2Enum)) (ap (c.2Emeasure\_2Emeasurable\_sets \\
& A.27a) V0m)))) \wedge (\forall V3m \in ty\_2Enum\_2Enum. (\forall V4n \in ty\_2Enum\_2Enum. \\
& ((\neg(V3m = V4n)) \Rightarrow (p (ap (ap (c.2Epred\_set\_2EDISJOINT \ A.27a) (ap \\
& V1f \ V3m)) (ap \ V1f \ V4n)))))))))) \Rightarrow ((ap (ap \ c.2Ereal\_2Esum (ap (ap \\
& (c.2Epair\_2E\_2C \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum) \ c.2Enum\_2E0) \\
& V2n)) (ap (ap (c.2Ecombin\_2Eo \ ty\_2Enum\_2Enum \ ty\_2Erealx\_2Ereal \\
& (2^{A.27a})) (ap (c.2Emeasure\_2Emeasure \ A.27a) V0m)) \ V1f)) = (ap \\
& (ap (c.2Emeasure\_2Emeasure \ A.27a) V0m) (ap (c.2Epred\_set\_2EBIGUNION \\
& A.27a) (ap (ap (c.2Epred\_set\_2EIMAGE \ ty\_2Enum\_2Enum (2^{A.27a})) \\
& V1f) (ap \ c.2Epred\_set\_2Ecount \ V2n)))))))))
\end{aligned}$$