

# thm\_2Emeasure\_2EALGEBRA\_\_FINITE\_\_UNION (TMGL9v2d4XVJMZCpyVxd3LPVHVT5i2Mu5SW)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (2)$$

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (3)$$

**Definition 10** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2E$   
 Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in ( \quad (4)$$

$$(2^{(2^{A-27a})})^{(ty\_2Epair\_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))})$$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))}) \quad (5)$$

**Definition 11** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap ($

**Definition 12** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1sts \in (2^{(2^{A-27a})})$

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

**Definition 14** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c$

**Definition 15** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 16** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))$

**Definition 17** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A-27a}).(ap (c$

**Definition 18** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c\_2Ebool\_2E\_21 2)$

**Definition 19** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$   
 of type  $\iota \Rightarrow \iota$ .

**Definition 20** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 21** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c\_2Epred\_set\_2E$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (12)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1s \in (2^{A\_27a}). (\forall V2t \in (2^{A\_27a}). ((p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) (ap (ap (c\_2Epred\_set\_2EINSERT A\_27a) V0x) V1s)) V2t)) \Leftrightarrow ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) V2t)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V1s) V2t)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(2^{A\_27a})}). ((p (ap V0P (c\_2Epred\_set\_2EEMPTY A\_27a))) \wedge (\forall V1s \in (2^{A\_27a}). (((p (ap (c\_2Epred\_set\_2EFINITE A\_27a) V1s)) \wedge (p (ap V0P V1s))) \Rightarrow (\forall V2e \in A\_27a. ((\neg(p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2e) V1s))) \Rightarrow (p (ap V0P (ap (ap (c\_2Epred\_set\_2EINSERT A\_27a) V2e) V1s)))))) \Rightarrow (\forall V3s \in (2^{A\_27a}). ((p (ap (c\_2Epred\_set\_2EFINITE A\_27a) V3s)) \Rightarrow (p (ap V0P V3s)))))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap (c\_2Epred\_set\_2EBIGUNION A\_27a) (c\_2Epred\_set\_2EEMPTY (2^{A\_27a}))) = (c\_2Epred\_set\_2EEMPTY A\_27a)) \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). (\forall V1P \in \\ (2^{(2^{A_{.27a}})}). ((\text{ap } (c_{.2E} \text{pred\_set\_2EBIGUNION } A_{.27a}) (\text{ap } (\text{ap} \\ (c_{.2E} \text{pred\_set\_2INSERT } (2^{A_{.27a}})) V0s) V1P)) = (\text{ap } (\text{ap } (c_{.2E} \text{pred\_set\_2EUNION} \\ A_{.27a}) V0s) (\text{ap } (c_{.2E} \text{pred\_set\_2EBIGUNION } A_{.27a}) V1P)))))) \end{aligned} \quad (17)$$

**Theorem 1**

$$\begin{aligned} \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0a \in (\text{ty\_2Epair\_2Eprod} \\ (2^{A_{.27a}}) (2^{(2^{A_{.27a}})})). (\forall V1c \in (2^{(2^{A_{.27a}})}). (((\text{p } ( \\ \text{ap } (c_{.2E} \text{measure\_2Ealgebra } A_{.27a}) V0a)) \wedge ((\text{p } (\text{ap } (c_{.2E} \text{pred\_set\_2EFINITE} \\ (2^{A_{.27a}})) V1c)) \wedge (\text{p } (\text{ap } (\text{ap } (c_{.2E} \text{pred\_set\_2ESUBSET } (2^{A_{.27a}})) \\ V1c) (\text{ap } (c_{.2E} \text{measure\_2Esubsets } A_{.27a}) V0a)))))) \Rightarrow (\text{p } (\text{ap } (\text{ap } (c_{.2E} \text{bool\_2EIN} \\ (2^{A_{.27a}})) (\text{ap } (c_{.2E} \text{pred\_set\_2EBIGUNION } A_{.27a}) V1c) (\text{ap } (c_{.2E} \text{measure\_2Esubsets} \\ A_{.27a}) V0a)))))) \end{aligned}$$