thm_2Emeasure_2EBOREL__MEASURABLE__SETS (TMKDfwoBN-WRj7N3u4T2eHcLTHanZXfofh6T)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{2}$$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Eextreal_2Eextreal__lt *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eextr$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 10 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 11 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{3}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{4}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{5}$$

Definition 13 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 14 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Definition 15 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Let $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \tag{6}$$

Definition 16 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 18 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 19 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{7}$$

Definition 20 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Definition 21 *We define* c_2Epred__set_2Ecountable *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_3F}$

Definition 22 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Let $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \tag{8}$$

Definition 23 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2I$

Definition 24 *We define* c_2Emeasure_2Esubset__class *to be* $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27}}$

Definition 25 *We define* c_2Emeasure_2Ealgebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2}$

Definition 26 *We define* c_2Emeasure_2Esigma__algebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_2}$

Definition 27 *We define* c_2Epred__set_2EBIGINTER *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 28 *We define* c_2Emeasure_2Esigma *to be* $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1st \in (2^{(2^{A\_27a})}).(ap\ ($

Definition 29 *We define* c_2Emeasure_2EBorel *to be* $(ap\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma}\ ty\_2Eextreal\_2Eextre$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{10}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V1x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V1x)\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V1x)\ V0c)))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel})))) \tag{11}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V1x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V1x)\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0c)\ V1x)))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel})))) \tag{12}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V1x \in ty\_2Eextreal\_2Eextreal.\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V1x)\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V1x)\ V0c)))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))) \tag{13}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V1x \in ty\_2Eextreal\_2Eextreal.\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V1x)\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V0c)\ V1x)))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))) \tag{14}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(\forall V1d \in ty\_2Eextreal\_2Eextreal.\ (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V2x \in ty\_2Eextreal\_2Eextreal.\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V2x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0c)\ V2x))\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V2x)\ V1d))))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))) \tag{15}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(\forall V1d \in ty\_2Eextreal\_2Eextreal.\ (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V2x \in ty\_2Eextreal\_2Eextreal.\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V2x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V0c)\ V2x))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V2x)\ V1d))))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))) \tag{16}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(\forall V1d \in ty\_2Eextreal\_2Eextreal.\ (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V2x \in ty\_2Eextreal\_2Eextreal.\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V2x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0c)\ V2x))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V2x)\ V1d))))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))) \tag{17}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(\forall V1d \in ty\_2Eextreal\_2Eextreal. (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V2x \in ty\_2Eextreal\_2Eextreal. (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V2x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V0c)\ V2x)) (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V2x)\ V1d))))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel})))))\tag{18}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Eextreal\_2Eextreal)\ V0c)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Eextreal\_2Eextreal))) (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))\tag{19}$$

Assume the following.

$$(\forall V0c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V1x \in ty\_2Eextreal\_2Eextreal. (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V1x)\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Eextreal\_2Eextreal)\ V1x)\ V0c)))))) (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))\tag{20}$$

Theorem 1

$$((\forall V0c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V1x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V1x)\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V1x)\ V0c)))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))\land$$

$$((\forall V2c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V3x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V3x)\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V2c)\ V3x)))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))\land$$

$$((\forall V4c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V5x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V5x)\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V4c)\ V5x)))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))\land$$

$$((\forall V6c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V7x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V7x)\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V7x)\ V6c)))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))\land$$

$$((\forall V8c \in ty\_2Eextreal\_2Eextreal.(\forall V9d \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V10x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V10x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V8c)\ V10x))\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V10x)\ V9d))))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel})))))\land$$

$$((\forall V11c \in ty\_2Eextreal\_2Eextreal.(\forall V12d \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V13x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V13x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V11c)\ V13x))\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V13x)\ V12d))))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel})))))\land$$

$$((\forall V14c \in ty\_2Eextreal\_2Eextreal.(\forall V15d \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V16x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V16x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V14c)\ V16x))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V16x)\ V15d))))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel})))))\land$$

$$((\forall V17c \in ty\_2Eextreal\_2Eextreal.(\forall V18d \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V19x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V19x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V17c)\ V19x))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V19x)\ V18d))))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel})))))\land$$

$$((\forall V20c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Eextreal\_2Eextreal)\ V20c)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Eextreal\_2Eextreal)))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel}))))\land$$

$$(\forall V21c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Eextreal\_2Eextreal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V22x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V22x)\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Eextreal\_2Eextreal)\ V22x)\ V21c))))))\ (ap\ (c\_2Emeasure\_2Esubsets\ ty\_2Eextreal\_2Eextreal)\ \mathsf{c\_2Emeasure\_2EBorel})))))))))))$$