

# thm\_2Emeasure\_2ECARATHEODORY\_LEMMA (TMdDwFP8iqUtyvmspryK37siMtTpFCozJjs)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) P)))$

**Definition 5** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 6** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 7** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \tag{3}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$



Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (11)$$

**Definition 20** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

**Definition 21** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 22** We define  $c\_2Earithmic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 23** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 24** We define  $c\_2Earithmic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (12)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (13)$$

**Definition 25** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Emin\_2E\_40\ t$

Let  $c\_2Erealax\_2Etreall\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

Let  $c\_2Erealax\_2Etreall\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (15)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (16)$$

**Definition 26** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 27** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. (ap\ c\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etreall\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (17)$$



Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (25)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in \\ ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27^a)})}) \end{aligned} \quad (26)$$

**Definition 36** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends \\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A-27^b})^{A-27^b}))})_{A\_27a})(A\_27a^{A-27^b})) \end{aligned} \quad (27)$$

**Definition 37** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 38** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets \\ A\_27a \in (((2^{A-27^a})(ty\_2Epair\_2Eprod\ (2^{A-27^a})\ (ty\_2Epair\_2Eprod\ (2^{(2^A-27^a)})\ (ty\_2Erealax\_2Ereal^{(2^A-27^a)})))) \end{aligned} \quad (28)$$

**Definition 39** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 40** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27^a}).\lambda V1t \in (2^{A-27^a}).(ap\ (c\_2$

**Definition 41** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27^a}).\lambda V1t \in (2^{A-27^a}).(ap\ (c\_2$

**Definition 42** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0P \in (2^{A-27^a}).\lambda V1Q \in (2^{A-27^b}).(ap\ (c\_2$

**Definition 43** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in \\ ((2^{A-27^a})(ty\_2Epair\_2Eprod\ (2^{A-27^a})\ (ty\_2Epair\_2Eprod\ (2^{(2^A-27^a)})\ (ty\_2Erealax\_2Ereal^{(2^A-27^a)})))) \end{aligned} \quad (29)$$

**Definition 44** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A-27^a})\ (ty\_2Epair\_2Eprod\ (2^{A-27^a})))$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ( \\ (2^{(2^{A-27^a})})(ty\_2Epair\_2Eprod\ (2^{A-27^a})\ (2^{(2^A-27^a)}))) \end{aligned} \quad (30)$$

**Definition 45** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

**Definition 46** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27a})$

**Definition 47** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E3F$

**Definition 48** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))})) \quad (31)$$

**Definition 49** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2E$

**Definition 50** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 51** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 52** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 53** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 54** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}).(ap (c\_2E$

**Definition 55** We define  $c\_2Eseq\_2Esummable$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}).(ap (c$

**Definition 56** We define  $c\_2Emeasure\_2Ecountably\_subadditive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 57** We define  $c\_2Emeasure\_2Eincreasing$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 58** We define  $c\_2Emeasure\_2Elambda\_system$  to be  $\lambda A\_27a : \iota.\lambda V0gen \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 59** We define  $c\_2Emeasure\_2Eouter\_measure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge ((p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))) \quad (40)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (41)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t))))) \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\forall V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x))))) \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ 2.(((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \vee (p\ V1Q))) \Leftrightarrow (\exists V3x \in \\ A\_27a.((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A\_27a}).(((p\ V0P) \vee (\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \\ A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ( \\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg( \\ p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in \\ A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).( \\ \forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c.2Ecombin\_2El \\ A\_27a)\ V0x) = V0x)) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}). (\forall V1y \in \\ (2^{(2^{A\_27a})}). ((ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}). (\forall V1sts \in \\ (2^{(2^{A\_27a})}). (\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). \\ ((ap\ (c\_2Emeasure\_2Em\_space\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ( \\ 2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))) \\ V0sp)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \\ V1sts)\ V2mu))) = V0sp)))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}). (\forall V1sts \in \\ (2^{(2^{A\_27a})}). (\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). \\ ((ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))) \\ V0sp)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \\ V1sts)\ V2mu))) = V1sts)))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}). (\forall V1sts \in \\ (2^{(2^{A\_27a})}). (\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). \\ ((ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A\_27a}) \\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))) \\ V0sp)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \\ V1sts)\ V2mu))) = V2mu)))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0g0 \in (ty\_2Epair\_2Eprod \\ (2^{A\_27a})\ (2^{(2^{A\_27a})})). (\forall V1lam \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). \\ (((p\ (ap\ (c\_2Emeasure\_2Ealgebra\ A\_27a)\ V0g0)) \wedge (p\ (ap\ (c\_2Emeasure\_2Epositive \\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})}) \\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a) \\ V0g0))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0g0))\ V1lam)))))) \Rightarrow (p\ (ap\ (c\_2Emeasure\_2Ealgebra \\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ (ap\ (c\_2Emeasure\_2Espace \\ A\_27a)\ V0g0))\ (ap\ (ap\ (c\_2Emeasure\_2Elambda\_system\ A\_27a)\ V0g0) \\ V1lam)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A\_27a})\ (2^{(2^{A\_27a})})).((p\ (ap\ (c\_2Emeasure\_2Esigma\_algebra \\
& \quad A\_27a)\ V0a)) \Leftrightarrow ((p\ (ap\ (c\_2Emeasure\_2Ealgebra\ A\_27a)\ V0a)) \wedge (\forall V1f \in \\
& \quad ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ ((2^{A\_27a})^{ty\_2Enum\_2Enum})) \\
& \quad V1f)\ (ap\ (ap\ (c\_2Epred\_set\_2EFUNSET\ ty\_2Enum\_2Enum\ (2^{A\_27a})) \\
& \quad (c\_2Epred\_set\_2EUNIV\ ty\_2Enum\_2Enum))\ (ap\ (c\_2Emeasure\_2Esubsets \\
& \quad A\_27a)\ V0a)))) \wedge (\forall V2m \in ty\_2Enum\_2Enum. (\forall V3n \in ty\_2Enum\_2Enum. \\
& \quad ((\neg(V2m = V3n)) \Rightarrow (p\ (ap\ (ap\ (c\_2Epred\_set\_2EDISJOINT\ A\_27a)\ (ap \\
& \quad V1f\ V2m))\ (ap\ V1f\ V3n)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a})) \\
& \quad (ap\ (c\_2Epred\_set\_2EBIGUNION\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Enum\_2Enum\ (2^{A\_27a}))\ V1f)\ (c\_2Epred\_set\_2EUNIV\ ty\_2Enum\_2Enum)))))) \\
& \quad (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0g0 \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A\_27a})\ (2^{(2^{A\_27a})})).(\forall V1lam \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). \\
& \quad ((p\ (ap\ (c\_2Emeasure\_2Epositive\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))) \\
& \quad (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0g0))\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ (ap\ (c\_2Emeasure\_2Esubsets \\
& \quad A\_27a)\ V0g0))\ V1lam)))) \Rightarrow (p\ (ap\ (c\_2Emeasure\_2Epositive\ A\_27a) \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})}) \\
& \quad (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a) \\
& \quad V0g0))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))) \\
& \quad (ap\ (ap\ (c\_2Emeasure\_2Elambda\_system\ A\_27a)\ V0g0)\ V1lam))\ V1lam))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A\_27a})\ (2^{(2^{A\_27a})})).((p\ (ap\ (c\_2Emeasure\_2Esigma\_algebra \\
& \quad A\_27a)\ V0a)) \Rightarrow (p\ (ap\ (c\_2Emeasure\_2Ealgebra\ A\_27a)\ V0a))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0gsig \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A.27a})\ (2^{(2^{A.27a})})).(\forall V1lam \in (ty\_2Erealax\_2Ereal^{(2^{A.27a})}). \\
& \quad (((p\ (ap\ (c\_2Emeasure\_2Esigma\_algebra\ A.27a)\ V0gsig)) \wedge (p\ (ap \\
& \quad (c\_2Emeasure\_2Eouter\_measure\_space\ A.27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad (2^{A.27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A.27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))) \\
& \quad (ap\ (c\_2Emeasure\_2Espace\ A.27a)\ V0gsig))\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad (2^{(2^{A.27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))\ (ap\ (c\_2Emeasure\_2Esubsets \\
& \quad A.27a)\ V0gsig))\ V1lam)))))) \Rightarrow (\forall V2f \in ((2^{A.27a})^{ty\_2Enum\_2Enum}). \\
& \quad (((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ ((2^{A.27a})^{ty\_2Enum\_2Enum}))\ V2f)\ ( \\
& \quad ap\ (ap\ (c\_2Epred\_set\_2EFUNSET\ ty\_2Enum\_2Enum\ (2^{A.27a}))\ (c\_2Epred\_set\_2EUNIV \\
& \quad ty\_2Enum\_2Enum))\ (ap\ (ap\ (c\_2Emeasure\_2Elambda\_system\ A.27a)\ \\
& \quad V0gsig)\ V1lam)))))) \wedge (\forall V3m \in ty\_2Enum\_2Enum. (\forall V4n \in \\
& \quad ty\_2Enum\_2Enum. ((\neg(V3m = V4n)) \Rightarrow (p\ (ap\ (ap\ (c\_2Epred\_set\_2EDISJOINT \\
& \quad A.27a)\ (ap\ V2f\ V3m))\ (ap\ V2f\ V4n)))))) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad (2^{A.27a}))\ (ap\ (c\_2Epred\_set\_2EBIGUNION\ A.27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Enum\_2Enum\ (2^{A.27a}))\ V2f)\ (c\_2Epred\_set\_2EUNIV\ ty\_2Enum\_2Enum)))))) \\
& \quad (ap\ (ap\ (c\_2Emeasure\_2Elambda\_system\ A.27a)\ V0gsig)\ V1lam))) \wedge \\
& \quad (p\ (ap\ (ap\ c\_2Eseq\_2Esums\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ ty\_2Enum\_2Enum \\
& \quad ty\_2Erealax\_2Ereal\ (2^{A.27a}))\ V1lam)\ V2f))\ (ap\ V1lam\ (ap\ (c\_2Epred\_set\_2EBIGUNION \\
& \quad A.27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ ty\_2Enum\_2Enum\ (2^{A.27a})) \\
& \quad V2f)\ (c\_2Epred\_set\_2EUNIV\ ty\_2Enum\_2Enum))))))))))))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (65)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (67)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (68)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee (\neg( \\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{73}$$

### Theorem 1

$$\begin{aligned}
& \forall A\_27a. nonempty \ A\_27a \Rightarrow (\forall V0gsig \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a}) \ (2^{(2^{A\_27a})})). (\forall V1lam \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). \\
& (((p \ (ap \ (c\_2Emeasure\_2Esigma\_algebra \ A\_27a) \ V0gsig)) \wedge (p \ (ap \\
& (c\_2Emeasure\_2Eouter\_measure\_space \ A\_27a) \ (ap \ (ap \ (c\_2Epair\_2E\_2C \\
& (2^{A\_27a}) \ (ty\_2Epair\_2Eprod \ (2^{(2^{A\_27a})}) \ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \\
& (ap \ (c\_2Emeasure\_2Espace \ A\_27a) \ V0gsig)) \ (ap \ (ap \ (c\_2Epair\_2E\_2C \\
& (2^{(2^{A\_27a})}) \ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \ (ap \ (c\_2Emeasure\_2Esubsets \\
& A\_27a) \ V0gsig)) \ V1lam)))) \Rightarrow (p \ (ap \ (c\_2Emeasure\_2Emeasure\_space \\
& A\_27a) \ (ap \ (ap \ (c\_2Epair\_2E\_2C \ (2^{A\_27a}) \ (ty\_2Epair\_2Eprod \ (2^{(2^{A\_27a})}) \\
& (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \ (ap \ (c\_2Emeasure\_2Espace \ A\_27a) \\
& V0gsig)) \ (ap \ (ap \ (c\_2Epair\_2E\_2C \ (2^{(2^{A\_27a})}) \ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \\
& (ap \ (ap \ (c\_2Emeasure\_2Elambda\_system \ A\_27a) \ V0gsig) \ V1lam)) \\
& V1lam))))))
\end{aligned}$$