thm_2Emeasure_2EFN__MINUS__CMUL (TMQcDYq2gFhnyyEraBzukPiWZQUqFeRVx7Y)

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Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (3)$$

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \quad (4)$$

Let $c\_2Eextreal\_2EPosInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \quad (5)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $c\_2Eextreal\_2ENegInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \quad (6)$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (7)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (8)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \quad (9)$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 5 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ ($c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{10}$$

Definition 6 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 9 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 10 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Eextreal\_2ENormal : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \tag{11}$$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{12}$$

Let $c\_2Eextreal\_2Eextreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_mul \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{13}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{14}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{15}$$

Definition 11 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 12 *We define* c_2Eextreal_2Eextreal__of__num *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Eextreal$

Definition 13 *We define* c_2Eextreal_2Eextreal__lt *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eext$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Emeasure_2Efn__plus *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).$

Let $c\_2Eextreal\_2Eextreal\_\_ainv : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_ainv \in (ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal}) \tag{16}$$

Definition 17 *We define* c_2Emeasure_2Efn__minus *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}$

Definition 18 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{21}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{29}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{30}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{31}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2a \in ty\_2Eextreal\_2Eextreal.(\forall V3v2 \in ty\_2Erealax\_2Ereal. (\forall V4v3 \in ty\_2Erealax\_2Ereal.(\forall V5v5 \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ c\_2Eextreal\_2ENormal\ V0x))\ (ap\ c\_2Eextreal\_2ENormal\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y))) \wedge (((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ c\_2Eextreal\_2ENegInf)\ V2a)) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ c\_2Eextreal\_2EPosInf)\ c\_2Eextreal\_2EPosInf)) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ c\_2Eextreal\_2ENormal\ V3v2))\ c\_2Eextreal\_2EPosInf)) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ c\_2Eextreal\_2EPosInf)\ c\_2Eextreal\_2ENegInf)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ c\_2Eextreal\_2ENormal\ V4v3))\ c\_2Eextreal\_2ENegInf)) \Leftrightarrow False) \wedge ((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ c\_2Eextreal\_2EPosInf)\ (ap\ c\_2Eextreal\_2ENormal\ V5v5))) \Leftrightarrow False)))))))))))) \tag{32}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V0x)\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) = (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))) \tag{33}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal.((p\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V0x)\ V1y)) \Rightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0x)\ V1y))))) \tag{34}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal.(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0x)\ V1y)) \land (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V1y)\ V0x))) \Leftrightarrow (V0x = V1y)))) \tag{35}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal.(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V0x)) \land (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V1y))) \Rightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V0x)\ V1y)))))) \tag{36}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal.(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0x)\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))) \land (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V1y)\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))) \Rightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V0x)\ V1y)))))) \tag{37}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal.(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V0x)) \land (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V1y)\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))) \Rightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V0x)\ V1y))\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))))) \tag{38}$$

Assume the following.

$$((ap\ c\_2Eextreal\_2Eextreal\_\_ainv\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) = (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) \tag{39}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(((ap\ c\_2Eextreal\_2Eextreal\_\_ainv\ V0x) = (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(V0x = (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))) \tag{40}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal.((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V0x)\ V1y) = (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V1y)\ V0x)))) \tag{41}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal.((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ (ap\ c\_2Eextreal\_2Eextreal\_\_ainv\ V0x))\ V1y) = (ap\ c\_2Eextreal\_2Eextreal\_\_ainv\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V0x)\ V1y))))) \tag{42}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal.((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V0x)\ (ap\ c\_2Eextreal\_2Eextreal\_\_ainv\ V1y)) = (ap\ c\_2Eextreal\_2Eextreal\_\_ainv\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V0x)\ V1y))))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{44}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{45}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{46}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{50}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{51}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{52}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{53}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{58}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).$$
$$(\forall V1c \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$(ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1c)) \Rightarrow ((ap\ (\mathsf{c\_2Emeasure\_2Efn\_\_minus}$$
$$A\_27a)\ (\lambda V2x \in A\_27a.(ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ (ap$$
$$c\_2Eextreal\_2ENormal\ V1c))\ (ap\ V0f\ V2x)))) = (\lambda V3x \in A\_27a.($$
$$ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ (ap\ c\_2Eextreal\_2ENormal\ V1c))$$
$$(ap\ (ap\ (\mathsf{c\_2Emeasure\_2Efn\_\_minus}\ A\_27a)\ V0f)\ V3x))))) \wedge ((p\ (ap$$
$$(ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1c)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \Rightarrow$$
$$((ap\ (\mathsf{c\_2Emeasure\_2Efn\_\_minus}\ A\_27a)\ (\lambda V4x \in A\_27a.(ap\ (ap$$
$$c\_2Eextreal\_2Eextreal\_\_mul\ (ap\ c\_2Eextreal\_2ENormal\ V1c))\ ($$
$$ap\ V0f\ V4x)))) = (\lambda V5x \in A\_27a.(ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul$$
$$(ap\ c\_2Eextreal\_2Eextreal\_\_ainv\ (ap\ c\_2Eextreal\_2ENormal\ V1c)))$$
$$(ap\ (ap\ (\mathsf{c\_2Emeasure\_2Efn\_\_plus}\ A\_27a)\ V0f)\ V5x))))))))$$