

thm_2Emeasure_2EIMAGE_SING (TMSQqx- unUXd416v5Q1kAfAy78aXHT4Ctma7)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2EIN$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 11 We define `c_2Epred_set_2EINSERT` to be $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1s \in (2^{A.27a}).(ap (c_2E$

Definition 12 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 13 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 14 We define `c_2Epred_set_2EIMAGE` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0f \in (A.27b^{A.27a}).\lambda V1s \in$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{5}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{6}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1a \in A.27a.((\exists V2x \in A.27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \tag{8}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in (2^{A.27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a.((p (ap (ap (c_2Ebool_2EIN A.27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A.27a) V2x) V1t))))))) \tag{9}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((p (ap (ap (c_2Ebool_2EIN A.27a) V0x) (ap (ap (c_2Epred_set_2EINSERT A.27a) V1y) (c_2Epred_set_2EEMPTY A.27a)))) \Leftrightarrow (V0x = V1y)))) \tag{10}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0y \in A.27b.(\forall V1s \in (2^{A.27a}).(\forall V2f \in (A.27b^{A.27a}).((p (ap (ap (c_2Ebool_2EIN A.27b) V0y) (ap (ap (c_2Epred_set_2EIMAGE A.27a A.27b) V2f) V1s)))) \Leftrightarrow (\exists V3x \in A.27a.((V0y = (ap V2f V3x)) \wedge (p (ap (ap (c_2Ebool_2EIN A.27a) V3x) V1s))))))) \tag{11}$$

Theorem 1

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1x \in A_{.27a}.((ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_{.27a}\ A_{.27b})\ V0f)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_{.27a})\ V1x)\ (c_2Epred_set_2EEMPTY\ A_{.27a}))) = (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_{.27b})\ (ap\ V0f\ V1x))\ (c_2Epred_set_2EEMPTY\ A_{.27b}))))))$$