

thm_2Emeasure_2EINCREASING_ADDITIVE_SUMMABLE
(TMKU-
VDhSsS7Q869hmPsd84GLJ8UAwrDTHXE)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2E_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 4 We define $c_2Ecombin_2E_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2E_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2E_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 6 We define $c_2Ebool_2E_2EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 8 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P)))$

Definition 9 We define $c_2Ebool_2E_2E5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))))$

Definition 10 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(3)

Definition 12 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2E21 2) (\lambda V0t \in 2. V0t))$.

Definition 13 We define c_2Ebool_2E2F to be $(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2. V0t))$.

Definition 14 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E2F))$.

Definition 15 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2E21 2) (\lambda V0t \in 2. V0t))$.

Definition 16 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2E21 2) (\lambda V0t \in 2. V0t))$.

Definition 17 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A_27a}). \lambda V1sts \in (2^{(2^{A_27a})})$.

Definition 18 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2F)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealx_2Ereal$$
(4)

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Emeasure_2Emeasure A_27a \in (ty_2Erealx_2Ereal^{(2^{A_27a})})^{(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})}))})$$
(5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(6)

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal})$$
(7)

Definition 19 We define c_2Emin_2E40 to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge P x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 20 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal. (ap (c_2Emin_2E40 (the (\lambda x. x \in A \wedge P x)))))$.

Let $c_2Erealx_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})$$
(8)

Definition 21 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal. \lambda V1T2 \in ty_2Erealx_2Ereal. (ap (c_2Emin_2E40 (the (\lambda x. x \in A \wedge P x))))$.

Definition 30 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (18)$$

Definition 31 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}))$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in \\ ((2^{A-27a})(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})})))) \end{aligned} \quad (19)$$

Definition 32 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}))$

Definition 33 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 34 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in (2^{A-27a})$

Definition 35 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in (A_27b^{A-27a})$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (21)$$

Definition 36 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ c_2Enum_2EREP_num)$

Definition 37 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ c_2Ebool_2E21))))$

Definition 38 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 39 We define $c_2Epred_set_2Ecount$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Epred_set_2EG\ c_2Ecount))$

Definition 40 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A-27a}).(ap\ (c_2Einsert_2E3C\ c_2Ebool_2E21\ c_2Ebool_2E3F\ c_2Ebool_2E21\ c_2Ebool_2E3F))$

Definition 41 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap\ (c_2Ebool_2E21\ c_2Ebool_2E3F\ c_2Ebool_2E21\ c_2Ebool_2E3F))$

Definition 42 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap\ (c_2Epred_set_2Ecount\ c_2Ebool_2E3F\ c_2Ebool_2E21\ c_2Ebool_2E3F))$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})(ty_2Epair_2Eprod\ ty_2Enum_2Enum)) \quad (22)$$

Definition 43 We define $c_2Earithmetic_2E3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 44 We define $c_2Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (23)$$

Definition 45 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$.

Definition 46 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 47 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 48 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (24)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (25)$$

Definition 49 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (26)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealax_2Ereal\ (ty_2Epair_2Eprod\ A_27a\ A_27a))}) \quad (27)$$

Definition 50 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \quad (28)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (29)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (30)$$

Definition 51 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$
 Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends \\ A_27a A_27b \in & (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})_{A_27a})_{(A_27a)^{A_27b}})) \end{aligned} \quad (31)$$

Definition 52 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 53 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 54 We define $c_2Eseq_2Esummable$ to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).(ap (c$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (36)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))) \quad (46)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))) \quad (47)$$

Assume the following.

$$\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27a^{A_27c}). (\forall V2x \in A_27c. ((ap\ (ap\ (ap\ (c.2Ecombin_2Eo\ A_27c\ A_27b\ A_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}). (\forall V1y \in (2^{(2^{A_27a})}). ((ap\ (c_2Emeasure_2Espace\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})}))\ V0x)\ V1y)) = V0x))) \quad (51)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}). (\forall V1y \in (2^{(2^{A_27a})}). ((ap\ (c_2Emeasure_2Esubsets\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})}))\ V0x)\ V1y)) = V1y))) \quad (52)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})). ((p\ (ap\ (c_2Emeasure_2Ealgebra\ A_27a)\ V0a)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ (ap\ (c_2Emeasure_2Espace\ A_27a)\ V0a))\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a)))))) \quad (53)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})). (\forall V1c \in (2^{(2^{A_27a})}). (((p\ (ap\ (c_2Emeasure_2Ealgebra\ A_27a)\ V0a)) \wedge ((p\ (ap\ (c_2Epred_set_2EFINITE\ (2^{A_27a})\ V1c)) \wedge (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ (2^{A_27a})\ V1c)\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ (ap\ (c_2Epred_set_2EBIGUNION\ A_27a)\ V1c))\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a)))))) \quad (54)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1f \in ((2^{A_27a})^{ty_2Enum_2Enum}). (\forall V2n \in ty_2Enum_2Enum. \\
& (((p (ap (c_2Emeasure_2Ealgebra\ A_27a) (ap (ap (c_2Epair_2E_2C \\
& (2^{A_27a}) (2^{(2^{A_27a})})) (ap (c_2Emeasure_2Em_space\ A_27a) \\
& V0m)) (ap (c_2Emeasure_2Emeasurable_sets\ A_27a) V0m)))))) \wedge ((\\
& p (ap (c_2Emeasure_2Epositive\ A_27a) V0m)) \wedge ((p (ap (c_2Emeasure_2Eadditive \\
& A_27a) V0m)) \wedge ((p (ap (ap (c_2Ebool_2EIN ((2^{A_27a})^{ty_2Enum_2Enum}) \\
& V1f) (ap (ap (c_2Epred_set_2EFUNSET\ ty_2Enum_2Enum) (2^{A_27a})) \\
& (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)) (ap (c_2Emeasure_2Emeasurable_sets \\
& A_27a) V0m)))))) \wedge (\forall V3m \in ty_2Enum_2Enum. (\forall V4n \in ty_2Enum_2Enum. \\
& ((\neg (V3m = V4n)) \Rightarrow (p (ap (ap (c_2Epred_set_2EDISJOINT\ A_27a) (ap \\
& V1f\ V3m)) (ap\ V1f\ V4n)))))) \Rightarrow ((ap (ap\ c_2Ereal_2Esum (ap (ap \\
& (c_2Epair_2E_2C\ ty_2Enum_2Enum\ ty_2Enum_2Enum) c_2Enum_2E0) \\
& V2n)) (ap (ap (c_2Ecombin_2Eo\ ty_2Enum_2Enum\ ty_2Erealax_2Ereal \\
& (2^{A_27a})) (ap (c_2Emeasure_2Emeasure\ A_27a) V0m)) V1f)) = (ap \\
& (ap (c_2Emeasure_2Emeasure\ A_27a) V0m) (ap (c_2Epred_set_2EBIGUNION \\
& A_27a) (ap (ap (c_2Epred_set_2EIMAGE\ ty_2Enum_2Enum) (2^{A_27a})) \\
& V1f) (ap\ c_2Epred_set_2Ecount\ V2n)))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (p (ap (ap (c_2Ebool_2EIN \\
A_27a) V0x) (c_2Epred_set_2EUNIV\ A_27a)))) \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\
& ((p (ap (ap (c_2Ebool_2EIN\ A_27b) V0y) (ap (ap (c_2Epred_set_2EIMAGE \\
& A_27a\ A_27b) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\
& (p (ap (ap (c_2Ebool_2EIN\ A_27a) V3x) V1s)))))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27a}). (\forall V2Q \in \\
& (2^{A_27b}). ((p (ap (ap (c_2Ebool_2EIN (A_27b^{A_27a}) V0f) (ap (ap \\
& (c_2Epred_set_2EFUNSET\ A_27a\ A_27b) V1P) V2Q))) \Leftrightarrow (\forall V3x \in \\
& A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) V3x) V1P)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN \\
& A_27b) (ap\ V0f\ V3x)) V2Q)))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0s \in (2^{A.27a}).((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a) \\ & \quad V0s)) \Rightarrow (\forall V1f \in (A.27b^{A.27a}).(p\ (ap\ (c.2Epred_set.2EFINITE \\ & \quad A.27b)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27b)\ V1f)\ V0s)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty.2Enum.2Enum.(\forall V1n \in ty.2Enum.2Enum.(\\ & \quad (p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Enum.2Enum)\ V0m)\ (ap\ c.2Epred_set.2Ecount \\ & \quad V1n)))) \Leftrightarrow (p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ V0m)\ V1n)))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty.2Enum.2Enum.(p\ (ap\ (c.2Epred_set.2EFINITE \\ & \quad ty.2Enum.2Enum)\ (ap\ c.2Epred_set.2Ecount\ V0n)))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1sos \in \\ & \quad (2^{(2^{A.27a})}).((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (ap\ (c.2Epred_set.2EBIGUNION \\ & \quad A.27a)\ V1sos)))) \Leftrightarrow (\exists V2s \in (2^{A.27a}).((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & \quad A.27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ V2s)\ V1sos)))))) \end{aligned} \quad (62)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (64)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (66)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (67)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{72}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). ((\forall V1n \in \\
& ty_2Enum_2Enum. (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap V0f V1n)))) \wedge (\exists V2x \in ty_2Erealax_2Ereal. \\
& (\forall V3n \in ty_2Enum_2Enum. (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) c_2Enum_2E0) V3n)) V0f)) V2x)))))) \Rightarrow (p (ap c_2Eseq_2Esumnable \\
& V0f)))
\end{aligned} \tag{78}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))))). \\
& (\forall V1f \in ((2^{A.27a})^{ty_2Enum_2Enum}).(((p\ (ap\ (c_2Emeasure_2Ealgebra \\
A.27a)\ (ap\ (ap\ (c_2Epair_2E.2C\ (2^{A.27a})\ (2^{(2^{A.27a})})))\ (ap\ (c_2Emeasure_2Em_space \\
A.27a)\ V0m))\ (ap\ (c_2Emeasure_2Emeasurable_sets\ A.27a)\ V0m)))))) \wedge \\
& ((p\ (ap\ (c_2Emeasure_2Epositive\ A.27a)\ V0m)) \wedge ((p\ (ap\ (c_2Emeasure_2Eincreasing \\
A.27a)\ V0m)) \wedge ((p\ (ap\ (c_2Emeasure_2Eadditive\ A.27a)\ V0m)) \wedge ((\\
& p\ (ap\ (ap\ (c_2Ebool_2EIN\ ((2^{A.27a})^{ty_2Enum_2Enum}))\ V1f)\ (ap\ (\\
ap\ (c_2Epred_set_2EFUNSET\ ty_2Enum_2Enum\ (2^{A.27a}))\ (c_2Epred_set_2EUNIV \\
ty_2Enum_2Enum))\ (ap\ (c_2Emeasure_2Emeasurable_sets\ A.27a)\ \\
V0m)))))) \wedge (\forall V2m \in ty_2Enum_2Enum. (\forall V3n \in ty_2Enum_2Enum. \\
& ((\neg(V2m = V3n)) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT\ A.27a)\ (ap \\
V1f\ V2m))\ (ap\ V1f\ V3n))))))))) \Rightarrow (p\ (ap\ c_2Eseq_2Esummable\ (ap \\
& (ap\ (c_2Ecombin_2Eo\ ty_2Enum_2Enum\ ty_2Erealax_2Ereal\ (2^{A.27a})) \\
(ap\ (c_2Emeasure_2Emeasure\ A.27a)\ V0m))\ V1f))))))
\end{aligned}$$