

thm_2Emeasure_2EINF__MEASURE__COUNTABLY__SUBADDIT (TMJeMaYKKnLrLijsyJGrgu5EdSxYx4Zjcab)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P)))$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (ty_2Erealax_2Ereal^{(2^{A_27a})})(ty_2Epair_2Eprod\ (2^{A_27a}))\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})}))\ (ty_2Erealax_2Ereal^{(2^{A_27a})}) \tag{3}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{4}$$

Definition 9 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{6}$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow q\ Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Emin_2E_3D_3D_3E\ V0t)\ V2t)\ V2t)\ V1t2)\ V0t1))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{7}$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealx_2Ereal^{(ty_2Erealx_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \tag{8}$$

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ (c_2Emin_2E_3D_3D_3E\ V0t)\ V0t)\ c_2Ebool_2E_21\ 2)\ V0t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{9}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{10}$$

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ V0m)$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x)) \text{ else } (ap\ P\ 0)$ of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0P)))$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap\ (c_2Eprim_rec_2E_3C\ V0m)\ V1n)$

Definition 19 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap\ (c_2Earithmetic_2E_3E\ V0m)\ V1n)$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V1t2)\ V0t1))$

Definition 21 We define $c_2Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (11)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax}) \quad (12)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (15)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 24 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (16)$$

Definition 25 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 26 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (17)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 27 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 28 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 30 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (19)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (20)$$

Definition 31 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (21)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}}) \end{aligned} \quad (22)$$

Definition 32 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \quad (23)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (24)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in \\ ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(A_27a)}})) \end{aligned} \quad (25)$$

Definition 33 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends \\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ ((2^{(A_27b)})^{A_27b}})}))_{A_27a})_{(A_27a)^{A_27b}}) \end{aligned} \quad (26)$$

Definition 34 We define $c_Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 35 We define c_Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty$

Definition 36 We define $c_Eseq_2Esuminf$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap (c_2E$

Definition 37 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 38 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).)(ap V1f V0x))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})) \end{aligned} \quad (27)$$

Definition 39 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 40 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_s$

Definition 41 We define $c_2Eseq_2Esummable$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap (c$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ A_27a \in ((2^{(2^{A_27a})})(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (28)$$

Definition 42 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in ($

Definition 43 We define $c_2Emeasure_2Ecountably_subadditive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 44 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 45 We define $c_2Emeasure_2Eincreasing$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})$

Definition 46 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 47 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 48 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Emin_2E.40\ ty_2Ereal$

Definition 49 We define c_2Ereal_2Ein to be $\lambda V0p \in (2^{ty_2Erealax_2Ereal}).(ap\ c_2Erealax_2Ereal_neg\ (ap\ c$

Definition 50 We define $c_2Emeasure_2Ein$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})$

Definition 51 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))))) \quad (29)$$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (30)$$

Definition 52 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2E$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (31)$$

Definition 53 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2E$

Definition 54 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A_27a}). \lambda V1sts \in (2^{(2^{A_27a})})$

Definition 55 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 56 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a})$

Definition 57 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a})$

Definition 58 We define $c_2Epred_set_2EPOW$ to be $\lambda A_27a : \iota. \lambda V0set \in (2^{A_27a}). (ap\ (c_2Epred_set_2E$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (32)$$

Definition 59 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (33)$$

Definition 60 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0P \in (2^{A_27a}). \lambda V1Q \in (2^{A_27b})$

Definition 61 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a})$

Definition 62 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B))$

Definition 63 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 64 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic$

Definition 65 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Erealax_2Etreax_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (34)$$

Definition 66 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Definition 67 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) ty_2Erealax_2Ereal) \quad (35)$$

Assume the following.

$$True \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg (p V0t)))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t) \Leftrightarrow (p V0t)))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{47}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{48}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{49}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x)))))) \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \tag{52}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A.27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p (\\ & \quad ap V1Q V4x))))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.((\forall V2x \in A.27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in \\ & \quad A.27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in \\ & \quad A.27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in \\ & \quad A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ & \quad A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).((\exists V2x \in A.27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ & \quad V0P) \wedge (\exists V3x \in A.27a.(p (ap V1Q V3x)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ & \quad V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (\\ & (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee \\ & \quad (p V0A)))) \end{aligned} \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (62)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (63)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (64)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}). (\forall V1a \in A_{27a}. ((\exists V2x \in A_{27a}. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (65)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow (\forall V0P \in ((2^{A_{27b}})^{A_{27a}}). ((\forall V1x \in A_{27a}. (\exists V2y \in A_{27b}. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b}^{A_{27a}}). (\forall V4x \in A_{27a}. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (66)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow \forall A_{27c}. \text{nonempty } A_{27c} \Rightarrow (\forall V0f \in (A_{27b}^{A_{27a}}). (\forall V1g \in (A_{27a}^{A_{27c}}). (\forall V2x \in A_{27c}. ((ap (ap (ap (c_{2Ecombin_2Eo} A_{27c} A_{27b} A_{27a}) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \quad (67)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((ap (c_{2Ecombin_2EI} A_{27a}) V0x) = V0x)) \quad (68)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0sp \in (2^{A_{27a}}). (\forall V1sts \in (2^{(2^{A_{27a}})}). (\forall V2mu \in (ty_{2Erealax_2Ereal} (2^{A_{27a}})). ((ap (c_{2Emeasure_2Emeasurable_sets} A_{27a}) (ap (ap (c_{2Epair_2E_2C} (2^{A_{27a}}) (ty_{2Epair_2Eprod} (2^{(2^{A_{27a}})}) (ty_{2Erealax_2Ereal} (2^{A_{27a}})))))) V0sp) (ap (ap (c_{2Epair_2E_2C} (2^{(2^{A_{27a}})}) (ty_{2Erealax_2Ereal} (2^{A_{27a}})))) V1sts) V2mu))) = V1sts)))) \quad (69)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0sp \in (2^{A-27a}). (\forall V1sts \in \\
& \quad (2^{(2^{A-27a})}). (\forall V2mu \in (ty_2Erealax_2Ereal^{(2^{A-27a})}). \\
& ((ap\ (c_2Emeasure_2Emeasure\ A.27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A-27a}) \\
& \quad (ty_2Epair_2Eprod\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})})))) \\
& V0sp)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})}) \\
& \quad V1sts)\ V2mu)))) = V2mu))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& \quad (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})}))). \\
& \quad (\forall V1s \in (2^{A-27a}). (\forall V2x \in ty_2Erealax_2Ereal. ((\\
& \quad (p\ (ap\ (c_2Emeasure_2Ealgebra\ A.27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\
& \quad \quad 2^{A-27a})\ (2^{(2^{A-27a})})))\ (ap\ (c_2Emeasure_2Em_space\ A.27a)\ V0m)) \\
& \quad \quad (ap\ (c_2Emeasure_2Emeasurable_sets\ A.27a)\ V0m)))) \wedge ((p\ (ap\ (\\
& \quad c_2Emeasure_2Epositive\ A.27a)\ V0m)) \wedge ((p\ (ap\ (c_2Emeasure_2Eincreasing \\
& \quad \quad A.27a)\ V0m)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V2x) \\
& \quad (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal \\
& \quad \quad 2)\ V3r)\ (ap\ (c_2Ebool_2E_3F\ ((2^{A-27a})^{ty_2Enum_2Enum})))\ (\lambda V4f \in \\
& \quad \quad ((2^{A-27a})^{ty_2Enum_2Enum}). (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap \\
& \quad (c_2Ebool_2EIN\ ((2^{A-27a})^{ty_2Enum_2Enum}))\ V4f)\ (ap\ (ap\ (c_2Epred_set_2EFUNSET \\
& \quad \quad ty_2Enum_2Enum\ (2^{A-27a})\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)) \\
& \quad (ap\ (c_2Emeasure_2Emeasurable_sets\ A.27a)\ V0m))))\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A.27a)\ V1s)\ (ap\ (c_2Epred_set_2EBIGUNION \\
& \quad \quad A.27a)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ ty_2Enum_2Enum\ (2^{A-27a}) \\
& \quad \quad V4f)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))))))\ (ap\ (ap\ c_2Eseq_2Esums \\
& \quad (ap\ (ap\ (c_2Ecombin_2Eo\ ty_2Enum_2Enum\ ty_2Erealax_2Ereal\ (2^{A-27a}) \\
& \quad \quad (ap\ (c_2Emeasure_2Emeasure\ A.27a)\ V0m))\ V4f))\ V3r))))))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ (ap\ (c_2Emeasure_2Einf_measure \\
& \quad \quad A.27a)\ V0m)\ V1s))\ V2x))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A.27a}) (ty_2Epair_2Eprod (2^{(2^{A.27a})}) (ty_2Erealax_2Ereal^{(2^{A.27a})}))))). \\
& (\forall V1s \in (2^{A.27a}). (\forall V2e \in ty_2Erealax_2Ereal. ((\\
& (p (ap (c_2Emeasure_2Ealgebra\ A.27a) (ap (ap (c_2Epair_2E_2C (\\
& 2^{A.27a}) (2^{(2^{A.27a})}) (ap (c_2Emeasure_2Em_space\ A.27a) V0m)) \\
& (ap (c_2Emeasure_2Emeasurable_sets\ A.27a) V0m)))))) \wedge ((p (ap (\\
& c_2Emeasure_2Epositive\ A.27a) V0m)) \wedge ((p (ap (ap\ c_2Erealax_2Ereal_lt \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) V2e)) \wedge (p (ap (ap (\\
& c_2Epred_set_2ESUBSET\ A.27a) V1s) (ap (c_2Emeasure_2Em_space \\
& A.27a) V0m)))))) \Rightarrow (\exists V3f \in ((2^{A.27a})^{ty_2Enum_2Enum}). (\\
& \exists V4l \in ty_2Erealax_2Ereal. ((p (ap (ap (c_2Ebool_2EIN (\\
& 2^{A.27a})^{ty_2Enum_2Enum})) V3f) (ap (ap (c_2Epred_set_2EFUNSET \\
& ty_2Enum_2Enum (2^{A.27a}) (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)) \\
& (ap (c_2Emeasure_2Emeasurable_sets\ A.27a) V0m)))))) \wedge ((p (ap (\\
& ap (c_2Epred_set_2ESUBSET\ A.27a) V1s) (ap (c_2Epred_set_2EBIGUNION \\
& A.27a) (ap (ap (c_2Epred_set_2EIMAGE\ ty_2Enum_2Enum (2^{A.27a}) \\
& V3f) (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)))))) \wedge ((\forall V5m \in \\
& ty_2Enum_2Enum. (\forall V6n \in ty_2Enum_2Enum. ((\neg (V5m = V6n)) \Rightarrow \\
& (p (ap (ap (c_2Epred_set_2EDISJOINT\ A.27a) (ap V3f V5m)) (ap V3f \\
& V6n)))))) \wedge ((p (ap (ap\ c_2Eseq_2Esums (ap (ap (c_2Ecombin_2Eo\ ty_2Enum_2Enum \\
& ty_2Erealax_2Ereal (2^{A.27a}) (ap (c_2Emeasure_2Emeasure\ A.27a) \\
& V0m)) V3f)) V4l)) \wedge (p (ap (ap\ c_2Ereal_2Ereal_lte\ V4l) (ap (ap\ c_2Erealax_2Ereal_add \\
& (ap (ap (c_2Emeasure_2Einf_measure\ A.27a) V0m) V1s)) V2e))))))))))))) \\
& (72)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2a \in A.27a. (\forall V3b \in \\
& A.27b. (((ap (ap (c_2Epair_2E_2C\ A.27a\ A.27b) V0x) V1y) = (ap (ap \\
& (c_2Epair_2E_2C\ A.27a\ A.27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\
& (73)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in (ty_2Epair_2Eprod\ A.27a\ A.27b). (\exists V1q \in A.27a. \\
& (\exists V2r \in A.27b. (V0x = (ap (ap (c_2Epair_2E_2C\ A.27a\ A.27b) \\
& V1q) V2r)))))) \\
& (74)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27c^{A.27b})^{A.27a}). (\forall V1x \in \\
& A.27a. (\forall V2y \in A.27b. ((ap (ap (c_2Epair_2EUNCURRY\ A.27a \\
& A.27b\ A.27c) V0f) (ap (ap (c_2Epair_2E_2C\ A.27a\ A.27b) V1x) V2y)) = \\
& (ap (ap V0f V1x) V2y)))))) \\
& (75)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A.27a\ 2)^{A.27b}).(\forall V1v \in \\ A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ A.27a\ A.27b)\ V0f)))) \Leftrightarrow (\exists V2x \in A.27b.((ap\ (ap\ (c_2Epair_2E_2C \\ A.27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \\ & \end{aligned} \tag{76}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A.27a)))) \tag{77}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0y \in A.27b.(\forall V1s \in (2^{A.27a}).(\forall V2f \in (A.27b^{A.27a}). \\ (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ A.27a\ A.27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A.27a.((V0y = (ap\ V2f\ V3x)) \wedge \\ (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V3x)\ V1s)))))) \\ & \end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0f \in (A.27b^{A.27a}).(\forall V1P \in (2^{A.27a}).(\forall V2Q \in \\ (2^{A.27b}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A.27b^{A.27a})\ V0f)\ (ap\ (ap \\ (c_2Epred_set_2EFUNSET\ A.27a\ A.27b)\ V1P)\ V2Q)))) \Leftrightarrow (\forall V3x \in \\ A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A.27b)\ (ap\ V0f\ V3x)\ V2Q)))))) \\ & \end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1sos \in \\ (2^{(2^{A.27a})}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V0x)\ (ap\ (c_2Epred_set_2EBIGUNION \\ A.27a)\ V1sos)))) \Leftrightarrow (\exists V2s \in (2^{A.27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A.27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a})\ V2s)\ V1sos)))))) \\ & \end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27b}).(\forall V2x \in \\ (ty_2Epair_2Eprod\ A.27a\ A.27b).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ A.27a\ A.27b)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A.27a\ A.27b)\ \\ V0P)\ V1Q)))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ (ap\ (c_2Epair_2EFST \\ A.27a\ A.27b)\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27b)\ (ap\ (c_2Epair_2ESND \\ A.27a\ A.27b)\ V2x))\ V1Q)))))) \\ & \end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0set \in (2^{A-27a}). (\forall V1e \in \\ & (2^{A-27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A-27a})\ V1e)\ (ap\ (c_2Epred_set_2EPOW \\ & A.27a)\ V0set))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A.27a)\ V1e) \\ & V0set)))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\ & (((p\ (ap\ (ap\ c_2Erealx_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0))\ V0x)) \wedge (p\ (ap\ (ap\ c_2Erealx_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0))\ V1y))) \Rightarrow (p\ (ap\ (ap\ c_2Erealx_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0))\ (ap\ (ap\ c_2Erealx_2Ereal_mul\ V0x)\ V1y)))))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. ((ap\ (ap\ c_2Erealx_2Ereal_mul \\ & V0x)\ (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) = V0x)) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\ & (\forall V2z \in ty_2Erealx_2Ereal. (((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (\\ & ap\ c_2Ereal_2Ereal_lte\ V0x)\ V2z)))))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\ & ((\forall V2e \in ty_2Erealx_2Ereal. ((p\ (ap\ (ap\ c_2Erealx_2Ereal_lt \\ & (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V2e)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & V0x)\ (ap\ (ap\ c_2Erealx_2Ereal_add\ V1y)\ V2e)))))) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & V0x)\ V1y)))))) \end{aligned} \quad (86)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (87)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (88)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (89)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (90)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (91)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (92)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (93)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (94)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (95)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (96)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (97)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (98)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (99)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (100)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (101)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1x0 \in \\ & ty_2Erealax_2Ereal.(\forall V2c \in ty_2Erealax_2Ereal.((p (ap \\ & (ap c_2Eseq_2Esums V0x) V1x0)) \Rightarrow (p (ap (ap c_2Eseq_2Esums (\lambda V3n \in \\ & ty_2Enum_2Enum.(ap (ap c_2Erealax_2Ereal_mul V2c) (ap V0x V3n)))) \\ & (ap (ap c_2Erealax_2Ereal_mul V2c) V1x0))))))) \end{aligned} \quad (102)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1x \in \\ & ty_2Erealax_2Ereal.((p (ap (ap c_2Eseq_2Esums V0f) V1x)) \Leftrightarrow ((p \\ & (ap c_2Eseq_2Esummable V0f)) \wedge ((ap c_2Eseq_2Esuminf V0f) = V1x)))))) \end{aligned} \quad (103)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(((\forall V1n \in \\ & ty_2Enum_2Enum.(p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0)) (ap V0f V1n)))) \wedge (p (ap c_2Eseq_2Esummable V0f))) \Rightarrow \\ & (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0)) (ap c_2Eseq_2Esuminf V0f)))))) \end{aligned} \quad (104)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1g \in \\ & (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(((p (ap c_2Eseq_2Esummable \\ & V0f)) \wedge (p (ap c_2Eseq_2Esummable V1g))) \Rightarrow ((p (ap c_2Eseq_2Esummable \\ & (\lambda V2n \in ty_2Enum_2Enum.(ap (ap c_2Erealax_2Ereal_add (ap \\ & V0f V2n)) (ap V1g V2n)))) \wedge ((ap (ap c_2Erealax_2Ereal_add (ap \\ & c_2Eseq_2Esuminf V0f)) (ap c_2Eseq_2Esuminf V1g)) = (ap c_2Eseq_2Esuminf \\ & (\lambda V3n \in ty_2Enum_2Enum.(ap (ap c_2Erealax_2Ereal_add (ap \\ & V0f V3n)) (ap V1g V3n)))))))))) \end{aligned} \quad (105)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\
& \quad (\forall V1g \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V2h \in \\
& \quad ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Enum_2Enum}). \\
& \quad \quad ((\forall V3m \in ty_2Enum_2Enum. (\forall V4n \in ty_2Enum_2Enum. \\
& \quad \quad \quad (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad \quad \quad c_2Enum_2E0))\ (ap\ (ap\ V0f\ V3m)\ V4n)))))) \wedge ((\forall V5n \in ty_2Enum_2Enum. \\
& \quad (p\ (ap\ (ap\ c_2Eseq_2Esums\ (ap\ V0f\ V5n))\ (ap\ V1g\ V5n)))) \wedge ((p\ (ap\ c_2Eseq_2Esummable \\
& \quad \quad V1g)) \wedge (p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& \quad \quad \quad ty_2Enum_2Enum\ ty_2Enum_2Enum))\ V2h)\ (c_2Epred_set_2EUNIV \\
& \quad \quad \quad ty_2Enum_2Enum))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Enum_2Enum)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))\ (c_2Epred_set_2EUNIV \\
& \quad \quad \quad ty_2Enum_2Enum)))))) \Rightarrow (p\ (ap\ (ap\ c_2Eseq_2Esums\ (ap\ (ap\ (c_2Ecombin_2Eo \\
& \quad \quad \quad ty_2Enum_2Enum\ ty_2Erealax_2Ereal\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Enum_2Enum))\ (ap\ (c_2Epair_2EUNCURRY\ ty_2Enum_2Enum\ ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Erealax_2Ereal)\ V0f))\ V2h))\ (ap\ c_2Eseq_2Esuminf\ V1g))))))
\end{aligned}
\tag{106}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ c_2Eseq_2Esums\ (\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Ereal_2Epow \\
& \quad (ap\ (ap\ c_2Ereal_2E2F\ (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))))) \\
& \quad \quad (ap\ (ap\ c_2Earithmetic_2E2B\ V0n)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))
\end{aligned}
\tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V1g \in \\
& \quad (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). ((\forall V2n \in ty_2Enum_2Enum. \\
& \quad \quad (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad \quad c_2Enum_2E0))\ (ap\ V0f\ V2n)))) \wedge ((p\ (ap\ c_2Eseq_2Esummable\ V1g)) \wedge \\
& \quad \quad (\forall V3n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad \quad \quad (ap\ V0f\ V3n))\ (ap\ V1g\ V3n)))))) \Rightarrow ((p\ (ap\ c_2Eseq_2Esummable\ V0f)) \wedge \\
& \quad \quad (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Eseq_2Esuminf\ V0f))\ (ap \\
& \quad \quad \quad c_2Eseq_2Esuminf\ V1g))))))
\end{aligned}
\tag{108}$$

Assume the following.

$$\begin{aligned}
& (\exists V0f \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Enum_2Enum}). \\
& \quad (p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& \quad \quad \quad ty_2Enum_2Enum\ ty_2Enum_2Enum))\ V0f)\ (c_2Epred_set_2EUNIV \\
& \quad \quad \quad ty_2Enum_2Enum))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Enum_2Enum)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))\ (c_2Epred_set_2EUNIV \\
& \quad \quad \quad ty_2Enum_2Enum))))))
\end{aligned}
\tag{109}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Erealx_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap (ap c_2Ereal_2Epow \\
& (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) \\
& V0n)))) \\
& \tag{110}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})}))). \\
& (((p (ap (c_2Emeasure_2Ealgebra A_27a) (ap (ap (c_2Epair_2E_2C \\
& (2^{A_27a}) (2^{(2^{A_27a})}))) (ap (c_2Emeasure_2Em_space A_27a \\
& V0m)) (ap (c_2Emeasure_2Emeasurable_sets A_27a) V0m)))) \wedge ((\\
& p (ap (c_2Emeasure_2Epositive A_27a) V0m)) \wedge (p (ap (c_2Emeasure_2Eincreasing \\
& A_27a) V0m)))) \Rightarrow (p (ap (c_2Emeasure_2Ecountably_subadditive \\
& A_27a) (ap (ap (c_2Epair_2E_2C (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})} \\
& (ty_2Erealx_2Ereal^{(2^{A_27a})}))) (ap (c_2Emeasure_2Em_space \\
& A_27a) V0m)) (ap (ap (c_2Epair_2E_2C (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})}))) \\
& (ap (c_2Epred_set_2EPOW A_27a) (ap (c_2Emeasure_2Em_space \\
& A_27a) V0m)))) (ap (c_2Emeasure_2Einf_measure A_27a) V0m))))))
\end{aligned}$$