

thm_2Emeasure_2EINF__MEASURE__POS
(TMKTVoZJ4q8uikz4R4AnhScvvF1Nwr7FSJ8)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 6 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (ty_2Erealax_2Ereal^{(2^{A_27a})})(ty_2Epair_2Eprod\ (2^{A_27a}))\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})}))\ (ty_2Erealax_2Ereal^{(2^{A_27a})}) \tag{3}$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{4}$$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 8 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{6}$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V0t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{7}$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ (V0x\ V1y))$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \tag{8}$$

Definition 13 We define $c_2Ebool_2E_2F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{9}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{10}$$

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ V0m))$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A)\ P)$ of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Ebool_2E_23F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0P)))$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ c_2Eprim_rec_2E_3C\ V0m\ V1n)$

Definition 19 We define $c_2Earithmic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ c_2Earithmic_2E_3E\ V0m\ V1n)$

Definition 20 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21) 2) (\lambda V2t \in$

Definition 21 We define $c_Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (11)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS}) \quad (12)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40) (t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (15)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 24 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (16)$$

Definition 25 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 26 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (17)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 27 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 28 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 30 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (19)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (20)$$

Definition 31 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (21)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}}) \end{aligned} \quad (22)$$

Definition 32 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)} \quad (23)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (24)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in \\ ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \end{aligned} \quad (25)$$

Definition 33 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A_27b})^{A_27b}}))_{A_27a})_{(A_27a)^{A_27b}}) \end{aligned} \quad (26)$$

Definition 34 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 35 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 36 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 37 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).ap\ V1f\ V0x))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b \in ((2^{A_27a})_{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (27)$$

Definition 38 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in$

Definition 39 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).ap\ (c_2Epred_set_2E$

Definition 40 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).ap\ ($

Definition 41 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 42 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).ap\ (c_2E$

Definition 43 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).ap\ ($

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ & A_27a \in (((2^{(2^{A_27a})})_{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))})) \end{aligned} \quad (28)$$

Definition 44 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in ($

Definition 45 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).ap\ (c_2Emin_2E_40\ ty_2Ereal$

Definition 46 We define c_2Ereal_2Einf to be $\lambda V0p \in (2^{ty_2Erealax_2Ereal}).ap\ c_2Erealax_2Ereal_neg\ (ap\ c$

Definition 47 We define $c_2Emeasure_2Einf_measure$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a}$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in (\quad (2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (29)$$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (30)$$

Definition 48 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \quad (31)$$

Definition 49 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Emeasure_2Epositive\ A_27a)\ V0s)\ V1t$

Definition 50 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Emeasure_2Epositive\ A_27a)\ V0s)\ V1t$

Definition 51 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 52 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (37)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))) \quad (45)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c.2Ecombin_2EI A_27a) V0x) = V0x)) \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}).(((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b \\ A_27b)\ (c_2Ecombin_2EI\ A_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo \\ A_27a\ A_27b\ A_27a)\ V0f)\ (c_2Ecombin_2EI\ A_27a)) = V0f))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}).(\forall V1y \in \\ (2^{(2^{A_27a})}).((ap\ (c_2Emeasure_2Espace\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\ (2^{A_27a})\ (2^{(2^{A_27a})}))\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}).(\forall V1y \in \\ (2^{(2^{A_27a})}).((ap\ (c_2Emeasure_2Esubsets\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\ (2^{A_27a})\ (2^{(2^{A_27a})}))\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\ (2^{A_27a})\ (2^{(2^{A_27a})})).((p\ (ap\ (c_2Emeasure_2Ealgebra\ A_27a) \\ V0a)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ (ap\ (c_2Emeasure_2Espace \\ A_27a)\ V0a))\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0m \in (\text{ty_2Epair_2Eprod} \\
& (2^{A_27a}) (\text{ty_2Epair_2Eprod } (2^{(2^{A_27a})}) (\text{ty_2Erealax_2Ereal}^{(2^{A_27a})}))). \\
& (\forall V1g \in (2^{A_27a}).(\forall V2s \in (2^{A_27a}).(((p (\text{ap } (c_2Emeasure_2Ealgebra \\
A_27a) (\text{ap } (\text{ap } (c_2Epair_2E_2C } (2^{A_27a}) (2^{(2^{A_27a})})) (\text{ap } (c_2Emeasure_2Em_space \\
A_27a) V0m)) (\text{ap } (c_2Emeasure_2E measurable_sets } A_27a) V0m)))))) \wedge \\
& ((p (\text{ap } (c_2Emeasure_2Epositive } A_27a) V0m)) \wedge ((p (\text{ap } (\text{ap } (c_2Ebool_2EIN \\
(2^{A_27a}) V2s) (\text{ap } (c_2Emeasure_2E measurable_sets } A_27a) V0m)))) \wedge \\
& (p (\text{ap } (\text{ap } (c_2Epred_set_2ESUBSET } A_27a) V1g) V2s)))))) \Rightarrow (p (\text{ap } \\
& (\text{ap } (c_2Ebool_2EIN } \text{ty_2Erealax_2Ereal}) (\text{ap } (\text{ap } (c_2Emeasure_2Emeasure \\
A_27a) V0m) V2s)) (\text{ap } (c_2Epred_set_2EGSPEC } \text{ty_2Erealax_2Ereal} \\
\text{ty_2Erealax_2Ereal}) (\lambda V3r \in \text{ty_2Erealax_2Ereal}.\text{ap } (\text{ap } (c_2Epair_2E_2C \\
\text{ty_2Erealax_2Ereal } 2) V3r) (\text{ap } (c_2Ebool_2E_3F } ((2^{A_27a})^{\text{ty_2Enum_2Enum}})) \\
& (\lambda V4f \in ((2^{A_27a})^{\text{ty_2Enum_2Enum}}).\text{ap } (\text{ap } c_2Ebool_2E_2F_5C \\
& (\text{ap } (\text{ap } (c_2Ebool_2EIN } ((2^{A_27a})^{\text{ty_2Enum_2Enum}})) V4f) (\text{ap } (\text{ap } \\
(c_2Epred_set_2EFUNSET } \text{ty_2Enum_2Enum } (2^{A_27a})) (c_2Epred_set_2EUNIV \\
\text{ty_2Enum_2Enum})) (\text{ap } (c_2Emeasure_2E measurable_sets } A_27a) \\
V0m)))) (\text{ap } (\text{ap } c_2Ebool_2E_2F_5C } (\text{ap } (c_2Ebool_2E_21 } \text{ty_2Enum_2Enum}) \\
& (\lambda V5m \in \text{ty_2Enum_2Enum}.\text{ap } (c_2Ebool_2E_21 } \text{ty_2Enum_2Enum}) \\
& (\lambda V6n \in \text{ty_2Enum_2Enum}.\text{ap } (\text{ap } c_2Emin_2E_3D_3D_3E } (\text{ap } c_2Ebool_2E_7E \\
& (\text{ap } (\text{ap } (c_2Emin_2E_3D } \text{ty_2Enum_2Enum}) V5m) V6n)))) (\text{ap } (\text{ap } (c_2Epred_set_2EDISJOINT \\
A_27a) (\text{ap } V4f } V5m)) (\text{ap } V4f } V6n)))))) (\text{ap } (\text{ap } c_2Ebool_2E_2F_5C \\
& (\text{ap } (\text{ap } (c_2Epred_set_2ESUBSET } A_27a) V1g) (\text{ap } (c_2Epred_set_2EBIGUNION \\
A_27a) (\text{ap } (\text{ap } (c_2Epred_set_2EIMAGE } \text{ty_2Enum_2Enum } (2^{A_27a}) \\
V4f) (c_2Epred_set_2EUNIV } \text{ty_2Enum_2Enum})))) (\text{ap } (\text{ap } c_2Eseq_2Esums \\
& (\text{ap } (\text{ap } (c_2Ecombin_2Eo } \text{ty_2Enum_2Enum } \text{ty_2Erealax_2Ereal } (2^{A_27a}) \\
& (\text{ap } (c_2Emeasure_2Emeasure } A_27a) V0m)) V4f)) V3r))))))))) \\
& (52)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1g \in (2^{A.27a}).(\forall V2x \in ty_2Erealax_2Ereal.((\\
& (p\ (ap\ (c_2Emeasure_2Ealgebra\ A.27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\
& 2^{A.27a})\ (2^{(2^{A.27a})}))\ (ap\ (c_2Emeasure_2Em_space\ A.27a)\ V0m))) \\
& (ap\ (c_2Emeasure_2Emeasurable_sets\ A.27a)\ V0m)))) \wedge ((p\ (ap\ (\\
& c_2Emeasure_2Epositive\ A.27a)\ V0m)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal)\ V2x)\ (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ (\lambda V3r \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal\ 2)\ V3r)\ (ap\ (c_2Ebool_2E_3F\ ((2^{A.27a})^{ty_2Enum_2Enum}))) \\
& (\lambda V4f \in ((2^{A.27a})^{ty_2Enum_2Enum}).(ap\ (ap\ c_2Ebool_2E_2F_5C \\
& (ap\ (ap\ (c_2Ebool_2EIN\ ((2^{A.27a})^{ty_2Enum_2Enum}))\ V4f)\ (ap\ (ap \\
& (c_2Epred_set_2EFUNSET\ ty_2Enum_2Enum\ (2^{A.27a}))\ (c_2Epred_set_2EUNIV \\
& ty_2Enum_2Enum)))\ (ap\ (c_2Emeasure_2Emeasurable_sets\ A.27a) \\
& V0m))))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (c_2Ebool_2E_21\ ty_2Enum_2Enum) \\
& (\lambda V5m \in ty_2Enum_2Enum.(ap\ (c_2Ebool_2E_21\ ty_2Enum_2Enum) \\
& (\lambda V6n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ (ap\ c_2Ebool_2E_7E \\
& (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Enum_2Enum)\ V5m)\ V6n))))\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT \\
& A.27a)\ (ap\ V4f\ V5m))\ (ap\ V4f\ V6n))))))\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A.27a)\ V1g)\ (ap\ (c_2Epred_set_2EBIGUNION \\
& A.27a)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ ty_2Enum_2Enum\ (2^{A.27a})) \\
& V4f)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))))\ (ap\ (ap\ c_2Eseq_2Esums \\
& (ap\ (ap\ (c_2Ecombin_2Eo\ ty_2Enum_2Enum\ ty_2Erealax_2Ereal\ (2^{A.27a})) \\
& (ap\ (c_2Emeasure_2Emeasure\ A.27a)\ V0m))\ V4f))\ V3r))))))\ (ap\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)\ V2x))))))
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{63}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{67}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (2^{ty_2Erealax_2Ereal}).(\forall V1r \in ty_2Erealax_2Ereal. \\
& ((\exists V2x \in ty_2Erealax_2Ereal.(p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V2x) V0p))) \wedge (\forall V3x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) V0p))) \Rightarrow (p (\\
& ap (ap c_2Ereal_2Ereal_lte V1r) V3x)))))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1r) (ap c_2Ereal_2Einf V0p))))))
\end{aligned} \tag{69}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal(2^{A_27a}))))). \\
& (\forall V1g \in (2^{A_27a}).(((p (ap (c_2Emeasure_2Ealgebra A_27a) \\
& (ap (ap (c_2Epair_2E_2C (2^{A_27a}) (2^{(2^{A_27a})})) (ap (c_2Emeasure_2Em_space \\
& A_27a) V0m)) (ap (c_2Emeasure_2Emeasurable_sets A_27a) V0m)))))) \wedge \\
& ((p (ap (c_2Emeasure_2Epositive A_27a) V0m)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET \\
& A_27a) V1g) (ap (c_2Emeasure_2Em_space A_27a) V0m)))))) \Rightarrow (p (ap \\
& (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& (ap (ap (c_2Emeasure_2Einf_measure A_27a) V0m) V1g))))))
\end{aligned}$$