thm_2Emeasure_2EIN__MEASURABLE__BOREL (TMJvu5rDVXofhob5QXBFyxBdwZ34TVgd3t9)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \tag{2}$$

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{3}$$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{4}$$

Definition 7 *We define* c_2Eextreal_2Eextreal__lt *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eextr$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{5}$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{6}$$

Definition 11 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 12 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Let $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \tag{7}$$

Definition 13 *We define* c_2Epred__set_2EFUNSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in ($

Definition 14 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 16 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 17 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{8}$$

Definition 18 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Definition 19 *We define* c_2Epred__set_2Ecountable *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_3F

Definition 20 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 21 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 22 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 23 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 24 *We define* c_2Emeasure_2Esubset__class *to be* $\lambda A\_27a:\iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a}}$

Definition 25 *We define* c_2Emeasure_2Ealgebra *to be* $\lambda A\_27a:\iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2}$

Definition 26 *We define* c_2Emeasure_2Esigma__algebra *to be* $\lambda A\_27a:\iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_2}$

Definition 27 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 28 *We define* c_2Epred__set_2EBIGINTER *to be* $\lambda A\_27a:\iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 29 *We define* c_2Emeasure_2Esigma *to be* $\lambda A\_27a:\iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1st \in (2^{(2^{A\_27a})}).(ap\ ($

Definition 30 *We define* c_2Emeasure_2EBorel *to be* $(ap\ (ap\ ($c_2Emeasure_2Esigma $ty\_2Eextreal\_2Eextre$

Definition 31 *We define* c_2Epred__set_2EPREIMAGE *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V$

Definition 32 *We define* c_2Emeasure_2Emeasurable *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0a \in (ty\_2Epair\_2Epro$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). (\forall V1a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eextreal\_2Eextreal^{A\_27a}))\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ ty\_2Eextreal\_2Eextreal)\ V1a)\ \mathsf{c\_2Emeasure\_2EBorel})))\Leftrightarrow((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V1a))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eextreal\_2Eextreal^{A\_27a}))\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ ty\_2Eextreal\_2Eextreal)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V1a))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Eextreal\_2Eextreal))))\wedge(\forall V2c \in ty\_2Eextreal\_2Eextreal. (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPREIMAGE}\ A\_27a\ ty\_2Eextreal\_2Eextreal)\ V0f)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Eextreal\_2Eextreal\ ty\_2Eextreal\_2Eextreal)\ (\lambda V3x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Eextreal\_2Eextreal\ 2)\ V3x)\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ V3x)\ V2c))))))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V1a)))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V1a)))))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b))\Leftrightarrow((V0x = V2a)\wedge(V1y = V3b))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t)\Leftrightarrow(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f)))\Leftrightarrow(\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1s \in (2^{A\_27b}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPREIMAGE}\ A\_27a\ A\_27b)\ V0f)\ V1s)))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0f\ V2x))\ V1s)))))) \tag{20}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). (\forall V1a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eextreal\_2Eextreal^{A\_27a}))\ V0f) (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ ty\_2Eextreal\_2Eextreal) V1a)\ \mathsf{c\_2Emeasure\_2EBorel}))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra} A\_27a)\ V1a)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eextreal\_2Eextreal^{A\_27a})) V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ ty\_2Eextreal\_2Eextreal) (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V1a))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV} ty\_2Eextreal\_2Eextreal)))) \wedge (\forall V2c \in ty\_2Eextreal\_2Eextreal. (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER} A\_27a)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V3x \in A\_27a. (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V3x)\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt} (ap\ V0f\ V3x))\ V2c)))))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V1a)))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V1a)))))))))$$