

thm_2Emeasure_2EIN_MEASURABLE_BOREL_ALL_MEASURE (TMa1ARjFhTvDkXHA48vUpt3NFT968bUAxiw)

October 26, 2020

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c_2Emeasure_2Emeasurable_sets\ A_{27a} \in ((2^{(2^{A_{27a}})}) (ty_2Epair_2Eprod\ (2^{A_{27a}})\ (ty_2Epair_2Eprod\ (2^{(2^{A_{27a}})})\ (ty_2Erealx_2Ereal^{(2^{A_{27a}})})))) \quad (3)$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c_2Emeasure_2Em_space\ A_{27a} \in ((2^{A_{27a}}) (ty_2Epair_2Eprod\ (2^{A_{27a}})\ (ty_2Epair_2Eprod\ (2^{(2^{A_{27a}})})\ (ty_2Erealx_2Ereal^{(2^{A_{27a}})})))) \quad (4)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A_{27a} : \iota.\lambda A_{27b} : \iota.\lambda A_{27c} : \iota.(\lambda V0f \in ((A_{27c}^{A_{27b}})^{A_{27a}}))$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A_{27a} : \iota.\lambda A_{27b} : \iota.\lambda A_{27c} : \iota.(\lambda V0f \in ((A_{27c}^{A_{27b}})^{A_{27a}}))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}})).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_{27a}})))$

Definition 6 We define $c_2Ecombin_2Eo$ to be $\lambda A_{27a} : \iota.\lambda A_{27b} : \iota.\lambda A_{27c} : \iota.(\lambda V0f \in (A_{27b}^{A_{27c}}).\lambda V1g$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (5)$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})_{ty_2Eextreal_2Eextreal}) \quad (6)$$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A_27a})})_{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})})) \quad (7)$$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})_{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})})) \quad (8)$$

Definition 7 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 10 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (9)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (10)$$

Definition 13 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 14 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epair_2E_2C\ V0s\ V1t))$

Definition 15 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EIN)$.

Definition 16 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (A_27b^{A_27a}).(ap\ (c_2Epair_2E_2C\ V0f\ V1s))$

Definition 17 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 P)))$

Definition 19 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A.\lambda P \in (2^{(2^{A-27a})}).(ap (c_2Epred_set_2E_3F P))$

Definition 20 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A.\lambda P \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Ebool_2E_3F P))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{11}$$

Definition 21 We define $c_2Epred_set_2EINJ$ to be $\lambda A.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in (2^{A-27a}).(ap (c_2Ebool_2E_3F P))$

Definition 22 We define $c_2Epred_set_2Ecountable$ to be $\lambda A.\lambda P \in (2^{A-27a}).(ap (c_2Ebool_2E_3F P))$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.(ap (c_2Ebool_2E_3F P)) (V2t t1 t2))))))$

Definition 24 We define $c_2Epred_set_2EUNION$ to be $\lambda A.\lambda P \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Ebool_2E_3F P))$

Definition 25 We define $c_2Epred_set_2EDIFF$ to be $\lambda A.\lambda P \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Ebool_2E_3F P))$

Definition 26 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A.\lambda P \in (2^{A-27a}).(ap (c_2Ebool_2E_3F P))$

Definition 27 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A.\lambda P \in (2^{A-27a}).\lambda V1sts \in (2^{(2^{A-27a})}).(ap (c_2Ebool_2E_3F P))$

Definition 28 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A.\lambda P \in (ty_2Epair_2Eprod (2^{A-27a}) (2^{(2^{A-27a})})).(ap (c_2Ebool_2E_3F P))$

Definition 29 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A.\lambda P \in (ty_2Epair_2Eprod (2^{A-27a}) (2^{(2^{A-27a})})).(ap (c_2Ebool_2E_3F P))$

Definition 30 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A.\lambda P \in (2^{(2^{A-27a})}).(ap (c_2Epred_set_2EBIGUNION P))$

Definition 31 We define $c_2Emeasure_2Esigma$ to be $\lambda A.\lambda P \in (2^{A-27a}).\lambda V1st \in (2^{(2^{A-27a})}).(ap (c_2Ebool_2E_3F P))$

Definition 32 We define $c_2Emeasure_2EBorel$ to be $(ap (ap (c_2Emeasure_2Esigma ty_2Eextreal_2Eextreal)))$

Definition 33 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1g \in (2^{A-27a}).(ap (c_2Ebool_2E_3F P))$

Definition 34 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A.\lambda A_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in (2^{A-27a}).(ap (c_2Ebool_2E_3F P))$

Definition 35 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A-27a}) (2^{(2^{A-27a})})).(ap (c_2Ebool_2E_3F P))$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (\\ & ap\ V1Q\ V4x)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & 2. (((\forall V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in \\ & A_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ & 2^{A_27a}). (((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in \\ & A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}). (\forall V1y \in (2^{(2^{A_27a})}). ((ap\ (c_2Emeasure_2Espace\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})}))\ V0x)\ V1y)) = V0x))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}). (\forall V1y \in (2^{(2^{A_27a})}). ((ap\ (c_2Emeasure_2Esubsets\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})}))\ V0x)\ V1y)) = V1y))) \quad (26)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\
& \quad (\forall V1a \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})}))).(((\\
& \quad \quad p\ (ap\ (ap\ (c.2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a}))\ V0f) \\
& \quad \quad (ap\ (ap\ (c.2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal \\
& \quad \quad \quad V1a)\ c.2Emeasure_2EBorel)))) \Rightarrow ((\forall V2c \in ty_2Eextreal_2Eextreal. \\
& \quad \quad (p\ (ap\ (ap\ (c.2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c.2Epred_set_2EINTER \\
& \quad \quad \quad A.27a)\ (ap\ (c.2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V3x \in A.27a. \\
& \quad \quad \quad (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ 2)\ V3x)\ (ap\ (ap\ c.2Eextreal_2Eextreal_lt \\
& \quad \quad \quad (ap\ V0f\ V3x))\ V2c))))))\ (ap\ (c.2Emeasure_2Espace\ A.27a)\ V1a)))\ (\\
& \quad \quad ap\ (c.2Emeasure_2Esubsets\ A.27a)\ V1a)))) \wedge ((\forall V4c \in ty_2Eextreal_2Eextreal. \\
& \quad \quad (p\ (ap\ (ap\ (c.2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c.2Epred_set_2EINTER \\
& \quad \quad \quad A.27a)\ (ap\ (c.2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V5x \in A.27a. \\
& \quad \quad \quad (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ 2)\ V5x)\ (ap\ (ap\ c.2Eextreal_2Eextreal_le \\
& \quad \quad \quad V4c)\ (ap\ V0f\ V5x))))))\ (ap\ (c.2Emeasure_2Espace\ A.27a)\ V1a)))\ (\\
& \quad \quad ap\ (c.2Emeasure_2Esubsets\ A.27a)\ V1a)))) \wedge ((\forall V6c \in ty_2Eextreal_2Eextreal. \\
& \quad \quad (p\ (ap\ (ap\ (c.2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c.2Epred_set_2EINTER \\
& \quad \quad \quad A.27a)\ (ap\ (c.2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V7x \in A.27a. \\
& \quad \quad \quad (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ 2)\ V7x)\ (ap\ (ap\ c.2Eextreal_2Eextreal_le \\
& \quad \quad \quad (ap\ V0f\ V7x))\ V6c))))))\ (ap\ (c.2Emeasure_2Espace\ A.27a)\ V1a)))\ (\\
& \quad \quad ap\ (c.2Emeasure_2Esubsets\ A.27a)\ V1a)))) \wedge ((\forall V8c \in ty_2Eextreal_2Eextreal. \\
& \quad \quad (p\ (ap\ (ap\ (c.2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c.2Epred_set_2EINTER \\
& \quad \quad \quad A.27a)\ (ap\ (c.2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V9x \in A.27a. \\
& \quad \quad \quad (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ 2)\ V9x)\ (ap\ (ap\ c.2Eextreal_2Eextreal_lt \\
& \quad \quad \quad V8c)\ (ap\ V0f\ V9x))))))\ (ap\ (c.2Emeasure_2Espace\ A.27a)\ V1a)))\ (\\
& \quad \quad ap\ (c.2Emeasure_2Esubsets\ A.27a)\ V1a)))) \wedge ((\forall V10c \in ty_2Eextreal_2Eextreal. \\
& \quad \quad (\forall V11d \in ty_2Eextreal_2Eextreal.(p\ (ap\ (ap\ (c.2Ebool_2EIN \\
& \quad \quad \quad (2^{A.27a}))\ (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ (ap\ (c.2Epred_set_2EGSPEC \\
& \quad \quad \quad A.27a\ A.27a)\ (\lambda V12x \in A.27a.(ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ 2) \\
& \quad \quad \quad V12x)\ (ap\ (ap\ c.2Ebool_2E_2F_5C\ (ap\ (ap\ c.2Eextreal_2Eextreal_lt \\
& \quad \quad \quad V10c)\ (ap\ V0f\ V12x)))\ (ap\ (ap\ c.2Eextreal_2Eextreal_lt\ (ap\ V0f \\
& \quad \quad \quad V12x))\ V11d))))))\ (ap\ (c.2Emeasure_2Espace\ A.27a)\ V1a)))\ (ap\ (\\
& \quad \quad c.2Emeasure_2Esubsets\ A.27a)\ V1a)))) \wedge ((\forall V13c \in ty_2Eextreal_2Eextreal. \\
& \quad \quad (\forall V14d \in ty_2Eextreal_2Eextreal.(p\ (ap\ (ap\ (c.2Ebool_2EIN \\
& \quad \quad \quad (2^{A.27a}))\ (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ (ap\ (c.2Epred_set_2EGSPEC \\
& \quad \quad \quad A.27a\ A.27a)\ (\lambda V15x \in A.27a.(ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ 2) \\
& \quad \quad \quad V15x)\ (ap\ (ap\ c.2Ebool_2E_2F_5C\ (ap\ (ap\ c.2Eextreal_2Eextreal_le \\
& \quad \quad \quad V13c)\ (ap\ V0f\ V15x)))\ (ap\ (ap\ c.2Eextreal_2Eextreal_lt\ (ap\ V0f \\
& \quad \quad \quad V15x))\ V14d))))))\ (ap\ (c.2Emeasure_2Espace\ A.27a)\ V1a)))\ (ap\ (\\
& \quad \quad c.2Emeasure_2Esubsets\ A.27a)\ V1a)))) \wedge ((\forall V16c \in ty_2Eextreal_2Eextreal. \\
& \quad \quad (\forall V17d \in ty_2Eextreal_2Eextreal.(p\ (ap\ (ap\ (c.2Ebool_2EIN \\
& \quad \quad \quad (2^{A.27a}))\ (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ (ap\ (c.2Epred_set_2EGSPEC \\
& \quad \quad \quad A.27a\ A.27a)\ (\lambda V18x \in A.27a.(ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ 2) \\
& \quad \quad \quad V18x)\ (ap\ (ap\ c.2Ebool_2E_2F_5C\ (ap\ (ap\ c.2Eextreal_2Eextreal_lt \\
& \quad \quad \quad V16c)\ (ap\ V0f\ V18x)))\ (ap\ (ap\ c.2Eextreal_2Eextreal_le\ (ap\ V0f \\
& \quad \quad \quad V18x))\ V17d))))))\ (ap\ (c.2Emeasure_2Espace\ A.27a)\ V1a)))\ (ap\ (\\
& \quad \quad c.2Emeasure_2Esubsets\ A.27a)\ V1a)))) \wedge ((\forall V19c \in ty_2Eextreal_2Eextreal. \\
& \quad \quad (\forall V20d \in ty_2Eextreal_2Eextreal.(p\ (ap\ (ap\ (c.2Ebool_2EIN \\
& \quad \quad \quad (2^{A.27a}))\ (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ (ap\ (c.2Epred_set_2EGSPEC \\
& \quad \quad \quad A.27a\ A.27a)\ (\lambda V21x \in A.27a.(ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ 2) \\
& \quad \quad \quad V21x)\ (ap\ (ap\ c.2Ebool_2E_2F_5C\ (ap\ (ap\ c.2Eextreal_2Eextreal_le \\
& \quad \quad \quad V19c)\ (ap\ V0f\ V21x)))\ (ap\ (ap\ c.2Eextreal_2Eextreal_le\ (ap\ V0f \\
& \quad \quad \quad V21x))\ V20d))))))\ (ap\ (c.2Emeasure_2Espace\ A.27a)\ V1a)))\ (ap\ (\\
& \quad \quad c.2Emeasure_2Esubsets\ A.27a)\ V1a)))) \wedge ((\forall V22c \in ty_2Eextreal_2Eextreal. \\
& \quad \quad (p\ (ap\ (ap\ (c.2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c.2Epred_set_2EINTER \\
& \quad \quad \quad A.27a)\ (ap\ (c.2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V23x \in A.27a. \\
& \quad \quad \quad (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ 2)\ V23x)\ (ap\ c.2Ebool_2E_7E\ (ap\ (\\
& \quad \quad \quad ap\ (c.2Emin_2E_3D\ ty_2Eextreal_2Eextreal)\ (ap\ V0f\ V23x))\ V22c))))))\ (\\
& \quad \quad ap\ (c.2Emeasure_2Espace\ A.27a)\ V1a)))\ (ap\ (c.2Emeasure_2Esubsets
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (41)$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\
& \quad (\forall V1m \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (ty_2Epair_2Eprod \\
& \quad \quad (2^{(2^{A.27a})})\ (ty_2Erealx_2Ereal^{(2^{A.27a})})))) \cdot ((p\ (ap\ (ap\ (\\
& c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a}))\ V0f)\ (ap\ (ap\ (c_2Emeasure_2E measurable \\
& \quad A.27a\ ty_2Eextreal_2Eextreal)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A.27a}) \\
& \quad \quad (2^{(2^{A.27a})})))\ (ap\ (c_2Emeasure_2Em_space\ A.27a)\ V1m))\ (ap\ (\\
& \quad c_2Emeasure_2E measurable_sets\ A.27a)\ V1m))))\ c_2Emeasure_2EBorel))) \Rightarrow \\
& \quad ((\forall V2c \in ty_2Eextreal_2Eextreal. (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A.27a)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad \quad A.27a\ A.27a)\ (\lambda V3x \in A.27a. (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2) \\
& \quad \quad V3x)\ (ap\ (ap\ c_2Eextreal_2Eextreal_lt\ (ap\ V0f\ V3x))\ V2c))))))\ (\\
& \quad ap\ (c_2Emeasure_2Em_space\ A.27a)\ V1m)))\ (ap\ (c_2Emeasure_2E measurable_sets \\
& \quad \quad A.27a)\ V1m)))) \wedge ((\forall V4c \in ty_2Eextreal_2Eextreal. (p\ (ap\ (\\
& \quad (ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad \quad A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V5x \in A.27a. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2)\ V5x)\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad \quad V4c)\ (ap\ V0f\ V5x))))))\ (ap\ (c_2Emeasure_2Em_space\ A.27a)\ V1m)))\ (\\
& \quad (ap\ (c_2Emeasure_2E measurable_sets\ A.27a)\ V1m)))) \wedge ((\forall V6c \in \\
& \quad ty_2Eextreal_2Eextreal. (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a})) \\
& \quad (ap\ (ap\ (c_2Epred_set_2EINTER\ A.27a)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad \quad A.27a\ A.27a)\ (\lambda V7x \in A.27a. (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2) \\
& \quad \quad V7x)\ (ap\ (ap\ c_2Eextreal_2Eextreal_le\ (ap\ V0f\ V7x))\ V6c))))))\ (\\
& \quad ap\ (c_2Emeasure_2Em_space\ A.27a)\ V1m)))\ (ap\ (c_2Emeasure_2E measurable_sets \\
& \quad \quad A.27a)\ V1m)))) \wedge ((\forall V8c \in ty_2Eextreal_2Eextreal. (p\ (ap\ (\\
& \quad (ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad \quad A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V9x \in A.27a. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2)\ V9x)\ (ap\ (ap\ c_2Eextreal_2Eextreal_lt \\
& \quad \quad V8c)\ (ap\ V0f\ V9x))))))\ (ap\ (c_2Emeasure_2Em_space\ A.27a)\ V1m)))\ (\\
& \quad (ap\ (c_2Emeasure_2E measurable_sets\ A.27a)\ V1m)))) \wedge ((\forall V10c \in \\
& \quad ty_2Eextreal_2Eextreal. (\forall V11d \in ty_2Eextreal_2Eextreal. \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad \quad A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V12x \in A.27a. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2)\ V12x)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad \quad (ap\ (ap\ c_2Eextreal_2Eextreal_lt\ V10c)\ (ap\ V0f\ V12x)))\ (ap\ (ap \\
& \quad c_2Eextreal_2Eextreal_lt\ (ap\ V0f\ V12x))\ V11d))))))\ (ap\ (c_2Emeasure_2Em_space \\
& \quad \quad A.27a)\ V1m)))\ (ap\ (c_2Emeasure_2E measurable_sets\ A.27a)\ V1m)))) \wedge \\
& \quad ((\forall V13c \in ty_2Eextreal_2Eextreal. (\forall V14d \in ty_2Eextreal_2Eextreal. \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad \quad A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V15x \in A.27a. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2)\ V15x)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad \quad (ap\ (ap\ c_2Eextreal_2Eextreal_le\ V13c)\ (ap\ V0f\ V15x)))\ (ap\ (ap \\
& \quad c_2Eextreal_2Eextreal_lt\ (ap\ V0f\ V15x))\ V14d))))))\ (ap\ (c_2Emeasure_2Em_space \\
& \quad \quad A.27a)\ V1m)))\ (ap\ (c_2Emeasure_2E measurable_sets\ A.27a)\ V1m)))) \wedge \\
& \quad ((\forall V16c \in ty_2Eextreal_2Eextreal. (\forall V17d \in ty_2Eextreal_2Eextreal. \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad \quad A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V18x \in A.27a. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2)\ V18x)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad \quad (ap\ (ap\ c_2Eextreal_2Eextreal_lt\ V16c)\ (ap\ V0f\ V18x)))\ (ap\ (ap \\
& \quad c_2Eextreal_2Eextreal_le\ (ap\ V0f\ V18x))\ V17d))))))\ (ap\ (c_2Emeasure_2Em_space \\
& \quad \quad A.27a)\ V1m)))\ (ap\ (c_2Emeasure_2E measurable_sets\ A.27a)\ V1m)))) \wedge \\
& \quad ((\forall V19c \in ty_2Eextreal_2Eextreal. (\forall V20d \in ty_2Eextreal_2Eextreal. \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad \quad A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V21x \in A.27a. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2)\ V21x)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad \quad (ap\ (ap\ c_2Eextreal_2Eextreal_le\ V19c)\ (ap\ V0f\ V21x)))\ (ap\ (ap \\
& \quad c_2Eextreal_2Eextreal_le\ (ap\ V0f\ V21x))\ V20d))))))\ (ap\ (c_2Emeasure_2Em_space \\
& \quad \quad A.27a)\ V1m)))\ (ap\ (c_2Emeasure_2E measurable_sets\ A.27a)\ V1m)))) \wedge \\
& \quad ((\forall V22c \in ty_2Eextreal_2Eextreal. (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A.27a)\ (ap\ (c_2Epred_set_2EGSPEC
\end{aligned}$$