

thm_2Emeasure_2EIN_MEASURABLE_BOREL_ALT3 (TMRGmp5YU7NMNjr6c2YiuSffuoSRhfkFdeB)

October 26, 2020

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0 f \in ((A.\lambda c^{A.\lambda b})^{A.\lambda a}))$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0 f \in ((A.\lambda c^{A.\lambda b})^{A.\lambda a}))$

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0 x \in A.\lambda a.(\lambda V1 y \in A.\lambda b.V0 x))$

Definition 6 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda a : \iota.(ap (ap (c_2Ecombin_2ES A.\lambda a (A.\lambda a^{A.\lambda a})) A.\lambda a))$

Definition 7 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0 x \in 2.V0 x)) (\lambda V1 x \in 2.V1 x))$

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0 P \in (2^{A.\lambda a}).(ap (ap (c_2Emin_2E_3D (2^{A.\lambda a})) P)))$

Definition 9 We define $c_2Ecombin_2Eo$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0 f \in (A.\lambda b^{A.\lambda c}).\lambda V1 g \in (A.\lambda a^{A.\lambda b}))$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \tag{2}$$

Definition 10 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0 t \in 2.V0 t))$.

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 12 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E$

Definition 13 We define $c_Eextreal_2Eextreal_It$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eext$

Definition 14 We define c_Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Definition 15 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 16 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (4)$$

Definition 17 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (5)$$

Definition 18 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 19 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2F)$.

Definition 20 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 21 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 22 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 23 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (6)$$

Definition 24 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2E)$.

Definition 25 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (2^{A$

Definition 26 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_3F$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in (\quad (7) \\ (2^{(2^A-27a)}) (ty_2Epair_2Eprod\ (2^A-27a)\ (2^{(2^A-27a)})))$$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^A-27a) (ty_2Epair_2Eprod\ (2^A-27a)\ (2^{(2^A-27a)}))) \quad (8)$$

Definition 27 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^A-27a). \lambda V1t \in (2^A-27a). (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ (V0s\ V1t))$

Definition 28 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^A-27a)\ (2^{(2^A-27a)}))$

Definition 29 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0P \in (2^A-27a). \lambda V1Q \in (2^A-27a)$

Definition 30 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^A-27a)\ (2^{(2^A-27a)}))$

Definition 31 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A-27a}). \lambda V1s \in (2^A-27a)$

Definition 32 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^A-27a)}). (ap\ (c_2Epred_set_2EDIFF\ A_27a)\ V0P)$

Definition 33 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^A-27a). \lambda V1st \in (2^{(2^A-27a)}). (ap\ (c_2Emeasure_2Esigma_algebra\ A_27a)\ V0sp\ V1st)$

Definition 34 We define $c_2Emeasure_2EBorel$ to be $(ap\ (ap\ (c_2Emeasure_2Esigma\ ty_2Eextreal_2Eextreal)\ V0sp)\ V1st)$

Definition 35 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A-27a}). \lambda V1s \in (2^A-27a)$

Definition 36 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^A-27a). \lambda V1t \in (2^A-27a). (ap\ (c_2Epred_set_2EBIGINTER\ A_27a)\ V0s\ V1t)$

Definition 37 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^A-27a)\ (2^{(2^A-27a)}))$

Definition 38 We define $c_2Epred_set_2ECOMPL$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^A-27a). (ap\ (ap\ (c_2Epred_set_2EBIGINTER\ A_27a)\ V0P))$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))
\end{aligned} \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\
& (2^{A_27a}).((\forall V2x \in A_27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\
& ((\forall V3x \in A_27a.(p \ (ap \ V0P \ V3x))) \wedge (\forall V4x \in A_27a.(p \ (\\
& ap \ V1Q \ V4x))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A_27a}).(((p \ V0P) \wedge (\forall V2x \in A_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in \\
& A_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A_27a}).((\forall V2x \in A_27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\
& V0P) \vee (\forall V3x \in A_27a.(p \ (ap \ V1Q \ V3x))))))
\end{aligned} \tag{21}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (24)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (25)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (26)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap (c_{.2Ecombin_2EI} A_{.27a}) V0x) = V0x)) \quad (27)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(((ap (ap (c_{.2Ecombin_2Eo} A_{.27a} A_{.27b} A_{.27b}) (c_{.2Ecombin_2EI} A_{.27b})) V0f) = V0f) \wedge ((ap (ap (c_{.2Ecombin_2Eo} A_{.27a} A_{.27b} A_{.27a}) V0f) (c_{.2Ecombin_2EI} A_{.27a})) = V0f))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\
& \quad (\forall V1a \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})})). ((\\
& \quad \quad p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a}))\ V0f) \\
& \quad \quad (ap\ (ap\ (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\
& \quad \quad V1a)\ c_2Emeasure_2EBorel))) \Leftrightarrow ((p\ (ap\ (c_2Emeasure_2Esigma_algebra \\
& \quad \quad A.27a)\ V1a)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a})) \\
& \quad \quad V0f)\ (ap\ (ap\ (c_2Epred_set_2EFUNSET\ A.27a\ ty_2Eextreal_2Eextreal) \\
& \quad \quad (ap\ (c_2Emeasure_2Espace\ A.27a)\ V1a))\ (c_2Epred_set_2EUNIV \\
& \quad \quad ty_2Eextreal_2Eextreal)))) \wedge (\forall V2c \in ty_2Eextreal_2Eextreal. \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad \quad A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V3x \in A.27a. \\
& \quad \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2)\ V3x)\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad \quad (ap\ V0f\ V3x))\ V2c))))))\ (ap\ (c_2Emeasure_2Espace\ A.27a)\ V1a))\ (\\
& \quad \quad ap\ (c_2Emeasure_2Esubsets\ A.27a)\ V1a)))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2a \in A.27a. (\forall V3b \in \\
& \quad \quad A.27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad \quad (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad \quad A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Epair_2Eprod\ A.27a\ 2)^{A.27b}). (\forall V1v \in \\
& \quad \quad A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad \quad A.27a\ A.27b)\ V0f))) \Leftrightarrow (\exists V2x \in A.27b. ((ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad A.27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). ((ap\ (ap\ (c_2Epred_set_2EDIFF\ A.27a)\ V0s)\ (ap\ (ap\ (\\
& \quad \quad c_2Epred_set_2EINTER\ A.27a)\ V1t)\ V0s)) = (ap\ (ap\ (c_2Epred_set_2EDIFF \\
& \quad \quad A.27a)\ V0s)\ V1t))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ & (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2Ecompl \\ & A_27a)\ V1s))) \Leftrightarrow (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V1s)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1t \in (2^{A_27b}). (\forall V2sp \in \\ & (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2Einter\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2Epreimage \\ & A_27a\ A_27b)\ V0f)\ (ap\ (c_2Epred_set_2Ecompl\ A_27b)\ V1t)))\ V2sp) = \\ & (ap\ (ap\ (c_2Epred_set_2Ediff\ A_27a)\ V2sp)\ (ap\ (ap\ (c_2Epred_set_2Epreimage \\ & A_27a\ A_27b)\ V0f)\ V1t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r)))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q))) \wedge (((p \vee V0p) \vee (\neg(p \vee V2r)))) \wedge (\\
& ((p \vee V1q) \vee ((p \vee V2r) \vee (\neg(p \vee V0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r)))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge (((p \vee V0p) \vee (\neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q) \vee ((p \vee V2r) \vee (\neg(p \vee V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow (\neg(p \vee V1q))) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge ((\neg(p \vee V1q) \vee (\neg(p \vee V0p))))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (\neg(p \vee V1q)))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow (\neg(p \vee V0p)))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow (\neg(p \vee V1q)))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \tag{50}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a. \text{nonempty } A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A-27a}). \\
& (\forall V1a \in (ty_2Epair_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))). ((\\
& p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A-27a})) V0f) \\
& (ap (ap (c_2Emeasure_2Emeasurable A.27a ty_2Eextreal_2Eextreal) \\
& V1a) c_2Emeasure_2EBorel))) \Leftrightarrow ((p (ap (c_2Emeasure_2Esigma_algebra \\
& A.27a) V1a)) \wedge ((p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A-27a})) \\
& V0f) (ap (ap (c_2Epred_set_2EFUNSET A.27a ty_2Eextreal_2Eextreal) \\
& (ap (c_2Emeasure_2Espace A.27a) V1a)) (c_2Epred_set_2EUNIV \\
& ty_2Eextreal_2Eextreal)))) \wedge (\forall V2c \in ty_2Eextreal_2Eextreal. \\
& (p (ap (ap (c_2Ebool_2EIN (2^{A-27a})) (ap (ap (c_2Epred_set_2EINTER \\
& A.27a) (ap (c_2Epred_set_2EGSPEC A.27a A.27a) (\lambda V3x \in A.27a. \\
& (ap (ap (c_2Epair_2E.2C A.27a 2) V3x) (ap (ap c_2Eextreal_2Eextreal_lt \\
& V2c) (ap V0f V3x)))))) (ap (c_2Emeasure_2Espace A.27a) V1a)) (\\
& ap (c_2Emeasure_2Esubsets A.27a) V1a))))))
\end{aligned}$$