

thm_2Emeasure_2EIN__MEASURABLE__BOREL__ALT4 (TMM7m2TCEFNf5WJCJekqjDKZBBwpNDLSAXR)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1f \in (2^{A_27a}))))$

Definition 7 We define $c_2Ecombin_2Eo$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27b^{A_27c})$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \tag{2}$$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \tag{3}$$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A_27a})).(ap V1f V0x))$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))))$

Definition 11 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (4)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A-27b})^{A-27a}}) \quad (5)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A-27b}}) \quad (6)$$

Definition 13 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap\ (c_2Epred_set_2E$

Definition 14 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2E$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (7)$$

Definition 15 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2T).$

Definition 16 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A-27a}). \lambda V1s \in (2^{A-27a}).$

Definition 17 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). (ap\ (c_2Ebool_2E_3F$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A-27a})})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))}) \quad (8)$$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (c_2Ebool_2E_21\ 2))))$

Definition 19 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2E$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))}) \quad (9)$$

Definition 20 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2. V0t)).$

Definition 21 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 22 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E_27E))$

Definition 23 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_7E)$.

Definition 24 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 25 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (10)$$

Definition 26 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal$

Definition 27 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b})$

Definition 28 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 29 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 30 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2E_27E))$

Definition 31 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1st \in (2^{(2^{A_27a})}).(ap (c_2Emeasure_2Esigma_algebra))$

Definition 32 We define $c_2Emeasure_2EBorel$ to be $(ap (ap (c_2Emeasure_2Esigma ty_2Eextreal_2Eextreal_le)))$

Definition 33 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 34 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E_27E))$

Definition 35 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \wedge (\forall V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A_27a.(p (ap V1Q V3x))))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (26)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (27)$$

Assume the following.

$$((\forall V0x \in ty_2Eextreal_2Eextreal.((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2ENegInf) V0x)) \wedge (p (ap (ap c_2Eextreal_2Eextreal_le V0x) c_2Eextreal_2EPosInf)))))) \wedge ((\forall V1x \in ty_2Eextreal_2Eextreal.((p (ap (ap c_2Eextreal_2Eextreal_le V1x) c_2Eextreal_2ENegInf)) \Leftrightarrow (V1x = c_2Eextreal_2ENegInf))) \wedge (\forall V2x \in ty_2Eextreal_2Eextreal.((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) V2x)) \Leftrightarrow (V2x = c_2Eextreal_2EPosInf)))))) \quad (28)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod (2^{A_{.27a}}) (2^{(2^{A_{.27a}})})).(\forall V1s \in (2^{A_{.27a}}).(\forall V2t \in (2^{A_{.27a}}).(((p (ap (c_2Emeasure_2Ealgebra A_{.27a}) V0a)) \wedge ((p (ap (ap (c_2Ebool_2EIN (2^{A_{.27a}}) V1s) (ap (c_2Emeasure_2Esubsets A_{.27a}) V0a))) \wedge (p (ap (ap (c_2Ebool_2EIN (2^{A_{.27a}}) V2t) (ap (c_2Emeasure_2Esubsets A_{.27a}) V0a)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN (2^{A_{.27a}}) (ap (ap (c_2Epred_set_2EINTER A_{.27a}) V1s) V2t)) (ap (c_2Emeasure_2Esubsets A_{.27a}) V0a))))))))) \quad (29)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\
& \quad (\forall V1a \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})}))). ((\\
& \quad \quad p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a}))\ V0f) \\
& \quad \quad (ap\ (ap\ (c_2Emeasure_2E measurable\ A.27a\ ty_2Eextreal_2Eextreal) \\
& \quad \quad V1a)\ c_2Emeasure_2EBorel))) \Leftrightarrow ((p\ (ap\ (c_2Emeasure_2Esigma_algebra \\
& \quad A.27a)\ V1a)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a})) \\
& \quad V0f)\ (ap\ (ap\ (c_2Epred_set_2EFUNSET\ A.27a\ ty_2Eextreal_2Eextreal) \\
& \quad \quad (ap\ (c_2Emeasure_2Espace\ A.27a)\ V1a))\ (c_2Epred_set_2EUNIV \\
& \quad \quad ty_2Eextreal_2Eextreal)))) \wedge (\forall V2c \in ty_2Eextreal_2Eextreal. \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V3x \in A.27a. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2)\ V3x)\ (ap\ (ap\ c_2Eextreal_2Eextreal_lt \\
& \quad \quad (ap\ V0f\ V3x))\ V2c))))))\ (ap\ (c_2Emeasure_2Espace\ A.27a)\ V1a)))\ (\\
& \quad \quad ap\ (c_2Emeasure_2Esubsets\ A.27a)\ V1a)))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\
& \quad (\forall V1a \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})}))). ((\\
& \quad \quad p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a}))\ V0f) \\
& \quad \quad (ap\ (ap\ (c_2Emeasure_2E measurable\ A.27a\ ty_2Eextreal_2Eextreal) \\
& \quad \quad V1a)\ c_2Emeasure_2EBorel))) \Leftrightarrow ((p\ (ap\ (c_2Emeasure_2Esigma_algebra \\
& \quad A.27a)\ V1a)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a})) \\
& \quad V0f)\ (ap\ (ap\ (c_2Epred_set_2EFUNSET\ A.27a\ ty_2Eextreal_2Eextreal) \\
& \quad \quad (ap\ (c_2Emeasure_2Espace\ A.27a)\ V1a))\ (c_2Epred_set_2EUNIV \\
& \quad \quad ty_2Eextreal_2Eextreal)))) \wedge (\forall V2c \in ty_2Eextreal_2Eextreal. \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A.27a\ A.27a)\ (\lambda V3x \in A.27a. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ 2)\ V3x)\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad \quad V2c)\ (ap\ V0f\ V3x))))))\ (ap\ (c_2Emeasure_2Espace\ A.27a)\ V1a)))\ (\\
& \quad \quad ap\ (c_2Emeasure_2Esubsets\ A.27a)\ V1a)))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2a \in A.27a. (\forall V3b \in \\
& \quad A.27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}).(\forall V1v \in \\ & A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b.((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ & (2^{A_27a}).(\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{50}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a. \text{nonempty } A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A-27a}). \\
& (\forall V1a \in (ty_2Epair_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))). ((\\
& p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A-27a})) V0f) \\
& (ap (ap (c_2Emeasure_2Emeasurable A.27a ty_2Eextreal_2Eextreal) \\
& V1a) c_2Emeasure_2EBorel))) \Leftrightarrow ((p (ap (c_2Emeasure_2Esigma_algebra \\
& A.27a) V1a)) \wedge ((p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A-27a})) \\
& V0f) (ap (ap (c_2Epred_set_2EFUNSET A.27a ty_2Eextreal_2Eextreal) \\
& (ap (c_2Emeasure_2Espace A.27a) V1a)) (c_2Epred_set_2EUNIV \\
& ty_2Eextreal_2Eextreal)))))) \wedge (\forall V2c \in ty_2Eextreal_2Eextreal. \\
& (\forall V3d \in ty_2Eextreal_2Eextreal. (p (ap (ap (c_2Ebool_2EIN \\
& (2^{A-27a})) (ap (ap (c_2Epred_set_2EINTER A.27a) (ap (c_2Epred_set_2EGSPEC \\
& A.27a A.27a) (\lambda V4x \in A.27a. (ap (ap (c_2Epair_2E_2C A.27a 2) \\
& V4x) (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Eextreal_2Eextreal_le \\
& V2c) (ap V0f V4x))) (ap (ap c_2Eextreal_2Eextreal_lt (ap V0f V4x)) \\
& V3d)))))) (ap (c_2Emeasure_2Espace A.27a) V1a))) (ap (c_2Emeasure_2Esubsets \\
& A.27a) V1a)))))))))
\end{aligned}$$