



**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (3)$$

**Definition 13** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 14** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 16** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 17** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$ .

**Definition 18** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 2) (\lambda V0t \in 2.V0t))))$ .

**Definition 19** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2EEMPTY : \iota \Rightarrow \iota \Rightarrow \iota))$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (4)$$

**Definition 20** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_21 2)$ .

**Definition 21** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27a})$ .

**Definition 22** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_3F : \iota \Rightarrow \iota \Rightarrow \iota))$ .

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in (2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))} \quad (5)$$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))}) \quad (6)$$

**Definition 23** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap (c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t)))$ .

**Definition 24** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_7E : \iota \Rightarrow \iota \Rightarrow \iota))$ .

**Definition 25** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$ .

Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \quad (7)$$

Let  $c\_2Eextreal\_2Eextreal\_le : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (8)$$

**Definition 26** We define  $c\_2Eextreal\_2Eextreal\_lt$  to be  $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eextreal\_2Eextreal$

**Definition 27** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a}))$

**Definition 28** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (A\_27b^{A\_27a})$

**Definition 29** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_set\_2EIMAGE)\ P)$

**Definition 30** We define  $c\_2Emeasure\_2Esigma$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1st \in (2^{(2^{A\_27a})}).(ap\ (c\_2Emeasure\_2Esigma\_algebra)\ sp)$

**Definition 31** We define  $c\_2Emeasure\_2EBorel$  to be  $(ap\ (ap\ (c\_2Emeasure\_2Esigma\ ty\_2Eextreal\_2Eextreal)\ ty\_2Eextreal\_2Eextreal))$

**Definition 32** We define  $c\_2Epred\_set\_2EPREIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1Q \in (A\_27b^{A\_27a})$

**Definition 33** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Emeasure\_2EBorel)\ s)$

**Definition 34** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A\_27a})$

**Definition 35** We define  $c\_2Emeasure\_2Emeasurable$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a}))$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (13)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (17)
\end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\
& (2^{A\_27a}).((\forall V2x \in A\_27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\
& ((\forall V3x \in A\_27a.(p \ (ap \ V0P \ V3x))) \wedge (\forall V4x \in A\_27a.(p \ ( \\
& ap \ V1Q \ V4x)))))) \quad (22)
\end{aligned}$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{.27a}}). ((p\ V0P) \wedge (\forall V2x \in A_{.27a}. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_{.27a}. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (23)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_{.27a}}). ((\forall V2x \in A_{.27a}. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A_{.27a}. (p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))))) \quad (24)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{.27a}}). ((\forall V2x \in A_{.27a}. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_{.27a}. (p\ (ap\ V1Q\ V3x)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B))))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (29)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_{.27})) \wedge ((p\ V1x_{.27}) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_{.27})))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_{.27}) \Rightarrow (p\ V3y_{.27})))))) \quad (30)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}). (\forall V1a \in A_{.27a}. ((\exists V2x \in A_{.27a}. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))) \quad (31)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A.27a}). \\
& \quad (\forall V1a \in (ty\_2Epair\_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})}))). (( \\
& \quad \quad p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A.27a}))\ V0f) \\
& \quad \quad (ap\ (ap\ (c\_2Emeasure\_2Emeasurable\ A.27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad \quad V1a)\ c\_2Emeasure\_2EBorel))) \Leftrightarrow ((p\ (ap\ (c\_2Emeasure\_2Esigma\_algebra \\
& \quad A.27a)\ V1a)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A.27a})) \\
& \quad V0f)\ (ap\ (ap\ (c\_2Epred\_set\_2EFUNSET\ A.27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad \quad (ap\ (c\_2Emeasure\_2Espace\ A.27a)\ V1a))\ (c\_2Epred\_set\_2EUNIV \\
& \quad \quad ty\_2Eextreal\_2Eextreal)))) \wedge (\forall V2c \in ty\_2Eextreal\_2Eextreal. \\
& \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER \\
& \quad A.27a)\ (ap\ (c\_2Epred\_set\_2EGSPEC\ A.27a\ A.27a)\ (\lambda V3x \in A.27a. \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ A.27a\ 2)\ V3x)\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt \\
& \quad \quad (ap\ V0f\ V3x))\ V2c))))))\ (ap\ (c\_2Emeasure\_2Espace\ A.27a)\ V1a)))\ ( \\
& \quad \quad ap\ (c\_2Emeasure\_2Esubsets\ A.27a)\ V1a)))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A.27a}). \\
& \quad (\forall V1a \in (ty\_2Epair\_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})}))). (( \\
& \quad \quad p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A.27a}))\ V0f) \\
& \quad \quad (ap\ (ap\ (c\_2Emeasure\_2Emeasurable\ A.27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad \quad V1a)\ c\_2Emeasure\_2EBorel))) \Rightarrow ((p\ (ap\ (c\_2Emeasure\_2Esigma\_algebra \\
& \quad A.27a)\ V1a)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A.27a})) \\
& \quad V0f)\ (ap\ (ap\ (c\_2Epred\_set\_2EFUNSET\ A.27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad \quad (ap\ (c\_2Emeasure\_2Espace\ A.27a)\ V1a))\ (c\_2Epred\_set\_2EUNIV \\
& \quad \quad ty\_2Eextreal\_2Eextreal)))) \wedge (\forall V2c \in ty\_2Eextreal\_2Eextreal. \\
& \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A.27a}))\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER \\
& \quad A.27a)\ (ap\ (c\_2Epred\_set\_2EGSPEC\ A.27a\ A.27a)\ (\lambda V3x \in A.27a. \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ A.27a\ 2)\ V3x)\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ ty\_2Eextreal\_2Eextreal) \\
& \quad \quad (ap\ V0f\ V3x))\ V2c))))))\ (ap\ (c\_2Emeasure\_2Espace\ A.27a)\ V1a)))\ ( \\
& \quad \quad ap\ (c\_2Emeasure\_2Esubsets\ A.27a)\ V1a)))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad \forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2a \in A.27a. (\forall V3b \in \\
& \quad A.27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c\_2Epair\_2E\_2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\ & \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A_{.27a}\ 2)^{A_{.27b}}).(\forall V1v \in \\ & A_{.27a}.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & \quad A_{.27a}\ A_{.27b})\ V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A_{.27a}\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ V0x)\ (c\_2Epred\_set\_2EUNIV\ A_{.27a})))) \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\ & (2^{A_{.27a}}).(\forall V2x \in A_{.27a}.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a}) \\ & V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A_{.27a})\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ ( \\ & (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A_{.27a})\ V2x)\ V1t)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\ & (2^{A_{.27a}}).(\forall V2x \in A_{.27a}.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a}) \\ & V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EDIFF\ A_{.27a})\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ ( \\ & ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A_{.27a})\ V2x)\ V1t)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\ & \quad \forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1P \in (2^{A_{.27a}}).(\forall V2Q \in \\ & (2^{A_{.27b}}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A_{.27b}^{A_{.27a}}))\ V0f)\ (ap\ (ap \\ & (c\_2Epred\_set\_2EFUNSET\ A_{.27a}\ A_{.27b})\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\ & A_{.27a}.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A_{.27b})\ (ap\ V0f\ V3x))\ V2Q)))))) \end{aligned} \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (43)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (53)$$



Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (55)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0f \in (\text{ty\_2Eextreal\_2Eextreal}^{A_{27a}}). \\ & \quad (\forall V1a \in (\text{ty\_2Epair\_2Eprod } (2^{A_{27a}}) (2^{(2^{A_{27a}})}))) . (( \\ & \quad \quad p (\text{ap } (\text{ap } (\text{c\_2Ebool\_2EIN } (\text{ty\_2Eextreal\_2Eextreal}^{A_{27a}})) V0f) \\ & \quad \quad (\text{ap } (\text{ap } (\text{c\_2Emeasure\_2Emeasurable } A_{27a} \text{ ty\_2Eextreal\_2Eextreal} \\ & \quad \quad V1a) \text{ c\_2Emeasure\_2EBorel}))) \Rightarrow ((p (\text{ap } (\text{c\_2Emeasure\_2Esigma\_algebra} \\ & \quad \quad A_{27a} V1a)) \wedge ((p (\text{ap } (\text{ap } (\text{c\_2Ebool\_2EIN } (\text{ty\_2Eextreal\_2Eextreal}^{A_{27a}})) \\ & \quad \quad V0f) (\text{ap } (\text{ap } (\text{c\_2Epred\_set\_2EFUNSET } A_{27a} \text{ ty\_2Eextreal\_2Eextreal} \\ & \quad \quad (\text{ap } (\text{c\_2Emeasure\_2Espace } A_{27a} V1a)) (\text{c\_2Epred\_set\_2EUNIV} \\ & \quad \quad \text{ty\_2Eextreal\_2Eextreal})))))) \wedge (\forall V2c \in \text{ty\_2Eextreal\_2Eextreal}. \\ & \quad \quad (p (\text{ap } (\text{ap } (\text{c\_2Ebool\_2EIN } (2^{A_{27a}})) (\text{ap } (\text{ap } (\text{c\_2Epred\_set\_2EINTER} \\ & \quad \quad A_{27a} (\text{ap } (\text{c\_2Epred\_set\_2EGSPEC } A_{27a} A_{27a})) (\lambda V3x \in A_{27a}. \\ & \quad \quad (\text{ap } (\text{ap } (\text{c\_2Epair\_2E\_2C } A_{27a} 2) V3x) (\text{ap } \text{c\_2Ebool\_2E\_7E } (\text{ap } (\text{ap} \\ & \quad \quad (\text{c\_2Emin\_2E\_3D } \text{ty\_2Eextreal\_2Eextreal} ) (\text{ap } V0f V3x)) V2c)))))) \\ & \quad \quad (\text{ap } (\text{c\_2Emeasure\_2Espace } A_{27a} V1a))) (\text{ap } (\text{c\_2Emeasure\_2Esubsets} \\ & \quad \quad A_{27a} V1a)))))))))) \end{aligned}$$