

thm_2Emeasure_2EIN__MEASURABLE__BOREL__CMUL (TMW63LGKu5u2N1JJpvmysDXLm2adzk55QsP)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** *(the* $(\lambda x.x \in A \wedge p)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x))$

Definition 7 We define $c_2Ecombin_2Eo$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27a^{A_27c}))$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{1}$$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{2}$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \tag{3}$$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \tag{4}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{5}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (7)$$

Definition 8 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal_2Eextreal_of_num\ n)$.

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (8)$$

Let $c_2Eextreal_2Eextreal_inv : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_inv \in (ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal}) \quad (9)$$

Definition 9 We define $c_2Eextreal_2Eextreal_div$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal_2Eextreal.(ap\ c_2Eextreal_2Eextreal_div\ x\ y)$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (12)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (13)$$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Emeasure_2Esubsets\ A.27a \in (2^{(2^{A.27a})})^{(ty_2Epair_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})}))} \quad (14)$$

Definition 11 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E.21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E.21))$.

Definition 14 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eext$

Definition 15 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 16 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (15)$$

Definition 17 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \end{aligned} \quad (16)$$

Definition 18 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 19 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 20 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 21 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set$

Definition 22 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 23 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A$

Definition 24 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_3F$

Definition 25 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 26 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Espace A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})))} \quad (17)$$

Definition 27 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 28 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 29 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})}$

Definition 30 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}$

Definition 31 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A-2}$

Definition 32 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c_2Epred_s$

Definition 33 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1st \in (2^{(2^{A-27a})}).(ap$

Definition 34 We define $c_2Emeasure_2EBorel$ to be $(ap (ap (c_2Emeasure_2Esigma ty_2Eextreal_2Eextre$

Definition 35 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V$

Definition 36 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c$

Definition 37 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in ($

Definition 38 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Epro$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (18)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax} \quad (19)$$

Definition 39 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (20)$$

Definition 40 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 41 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (\\ & ap\ V1Q\ V4x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & 2. (((\forall V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in \\ & A_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q)))))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (39)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))) \quad (41)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2a \in ty_2Eextreal_2Eextreal. (\forall V3v2 \in ty_2Erealax_2Ereal. \\
& (\forall V4v3 \in ty_2Erealax_2Ereal. (\forall V5v5 \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2ENormal \\
& V0x)) (ap c_2Eextreal_2ENormal V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y))) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2ENegInf) \\
& V2a)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& c_2Eextreal_2EPosInf)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2ENormal V3v2)) c_2Eextreal_2EPosInf)) \Leftrightarrow True) \wedge \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& c_2Eextreal_2ENegInf)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2ENormal V4v3)) c_2Eextreal_2ENegInf)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& (ap c_2Eextreal_2ENormal V5v5))) \Leftrightarrow False)))))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. ((ap (ap c_2Eextreal_2Eextreal_mul \\
& (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) V0x) = (ap c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Eextreal_2Eextreal_lt (ap c_2Eextreal_2ENormal \\
& V0x)) (ap c_2Eextreal_2ENormal V1y))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt \\
& V0x) V1y))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. (\forall V1y \in ty_2Eextreal_2Eextreal. \\
& ((ap (ap c_2Eextreal_2Eextreal_mul V0x) V1y) = (ap (ap c_2Eextreal_2Eextreal_mul \\
& V1y) V0x))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. (\forall V1y \in ty_2Eextreal_2Eextreal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V2z)) \Rightarrow ((p (ap (ap \\
& c_2Eextreal_2Eextreal_lt V0x) (ap (ap c_2Eextreal_2Eextreal_div \\
& V1y) (ap c_2Eextreal_2ENormal V2z)))) \Leftrightarrow (p (ap (ap c_2Eextreal_2Eextreal_lt \\
& (ap (ap c_2Eextreal_2Eextreal_mul V0x) (ap c_2Eextreal_2ENormal \\
& V2z))) V1y))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. (\forall V1y \in ty_2Eextreal_2Eextreal. \\
& \quad (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad V2z) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \Rightarrow ((p (ap (ap \\
& \quad c_2Eextreal_2Eextreal_lt (ap (ap c_2Eextreal_2Eextreal_div \\
V1y) (ap c_2Eextreal_2ENormal V2z))) V0x) \Leftrightarrow (p (ap (ap c_2Eextreal_2Eextreal_lt \\
& \quad (ap (ap c_2Eextreal_2Eextreal_mul V0x) (ap c_2Eextreal_2ENormal \\
& \quad V2z))) V1y))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A_27a}). \\
& \quad (\forall V1a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))). ((\\
& \quad p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A_27a})) V0f) \\
& \quad (ap (ap (c_2Emeasure_2Emeasurable A_27a ty_2Eextreal_2Eextreal) \\
V1a) c_2Emeasure_2EBorel))) \Leftrightarrow ((p (ap (c_2Emeasure_2Esigma_algebra \\
A_27a) V1a)) \wedge ((p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A_27a})) \\
V0f) (ap (ap (c_2Epred_set_2EFUNSET A_27a ty_2Eextreal_2Eextreal) \\
& \quad (ap (c_2Emeasure_2Espace A_27a) V1a)) (c_2Epred_set_2EUNIV \\
& \quad ty_2Eextreal_2Eextreal)))) \wedge (\forall V2c \in ty_2Eextreal_2Eextreal. \\
& \quad (p (ap (ap (c_2Ebool_2EIN (2^{A_27a})) (ap (ap (c_2Epred_set_2EINTER \\
A_27a) (ap (c_2Epred_set_2EGSPEC A_27a A_27a) (\lambda V3x \in A_27a. \\
& \quad (ap (ap (c_2Epair_2E.2C A_27a 2) V3x) (ap (ap c_2Eextreal_2Eextreal_lt \\
& \quad (ap V0f V3x) V2c)))))) (ap (c_2Emeasure_2Espace A_27a) V1a))) (\\
& \quad ap (c_2Emeasure_2Esubsets A_27a) V1a))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\
& \quad (2^{A_27a})\ (2^{(2^{A_27a})})). (\forall V1k \in ty_2Eextreal_2Eextreal. \\
& (\forall V2f \in (ty_2Eextreal_2Eextreal^{A_27a}). ((p\ (ap\ (c_2Emeasure_2Esigma_algebra \\
& \quad A_27a)\ V0a)) \wedge (\forall V3x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& \quad V3x)\ (ap\ (c_2Emeasure_2Espace\ A_27a)\ V0a))) \Rightarrow ((ap\ V2f\ V3x) = V1k)))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A_27a}))\ V2f) \\
& \quad (ap\ (ap\ (c_2Emeasure_2E measurable\ A_27a\ ty_2Eextreal_2Eextreal \\
& \quad V0a)\ c_2Emeasure_2EBorel))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& \quad A_27b. ((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\
& \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A_27a))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& \quad V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V2x)\ V1t))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27a}). (\forall V2Q \in \\ & \quad (2^{A_27b}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A_27b^{A_27a})\ V0f)\ (ap\ (ap \\ & \quad (c_2Epred_set_2EFUNSET\ A_27a\ A_27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\ & \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27b)\ (ap\ V0f\ V3x))\ V2Q))))))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\ & ((p\ (ap\ (ap\ c_2Erealax_2Ereal_lte\ V0x)\ V1y)) \Leftrightarrow ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & \quad V0x)\ V1y)) \wedge (\neg(V0x = V1y)))))) \end{aligned} \quad (57)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (62)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee (\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q)) \vee (\neg(p\ V2r))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{67}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{72}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (2^{(2^{A_27a})})). (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}). \\
& (\forall V2g \in (ty_2Eextreal_2Eextreal^{A_27a}). (\forall V3z \in ty_2Erealax_2Ereal. \\
& (((p \ (ap \ (c_2Emeasure_2Esigma_algebra \ A_27a) \ V0a)) \wedge ((p \ (ap \ (\\
& ap \ (c_2Ebool_2EIN \ (ty_2Eextreal_2Eextreal^{A_27a}) \ V1f) \ (ap \ (ap \\
& (c_2Emeasure_2Emeasurable \ A_27a \ ty_2Eextreal_2Eextreal) \ V0a) \\
& c_2Emeasure_2EBorel)))) \wedge (\forall V4x \in A_27a. ((p \ (ap \ (ap \ (c_2Ebool_2EIN \\
& A_27a) \ V4x) \ (ap \ (c_2Emeasure_2Espace \ A_27a) \ V0a))) \Rightarrow ((ap \ V2g \ V4x) = \\
& (ap \ (ap \ c_2Eextreal_2Eextreal_mul \ (ap \ c_2Eextreal_2ENormal \\
& V3z)) \ (ap \ V1f \ V4x)))))) \Rightarrow (p \ (ap \ (ap \ (c_2Ebool_2EIN \ (ty_2Eextreal_2Eextreal^{A_27a}) \\
& V2g) \ (ap \ (ap \ (c_2Emeasure_2Emeasurable \ A_27a \ ty_2Eextreal_2Eextreal) \\
& V0a) \ c_2Emeasure_2EBorel))))))
\end{aligned}$$