

thm_2Emeasure_2EIN__MEASURABLE__BOREL__LE (TML7rrLsnLjFPEAgW8hYdd8Zq5fd1WhNf7J)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 5 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 7 We define $c_2Ecombin_2Eo$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \tag{2}$$

Definition 8 We define $c_2Ebool_2E_2EIN$ to be $\lambda A.\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 10 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t)))$

Definition 11 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (4)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ x\ y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2^{A_27b})}) \quad (5)$$

Definition 13 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ s\ t)$

Definition 14 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 16 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ s\ t)$

Definition 17 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})^{A_27a}})$

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ P)))$

Definition 19 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})^{A_27a}}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ P)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (6)$$

Definition 20 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 21 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 22 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_3F\ A_27a)\ s)$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A_27a})^{A_27a}})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})^{A_27a}}))}) \quad (7)$$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})^{A_27a}}))}) \quad (8)$$

Definition 23 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 24 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 25 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{2^{A_27a}}$

Definition 26 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{2^{A_27a}}$

Definition 27 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{2^{A_27a}}$

Definition 28 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal$

Definition 29 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 30 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2E$

Definition 31 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1st \in (2^{(2^{A_27a})}).(ap ($

Definition 32 We define $c_2Emeasure_2EBorel$ to be $(ap (ap (c_2Emeasure_2Esigma ty_2Eextreal_2Eextreal$

Definition 33 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 34 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 35 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Eprod$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{11}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{17}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\
& (2^{A_27a}).((\forall V2x \in A_27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\
& ((\forall V3x \in A_27a.(p \ (ap \ V0P \ V3x))) \wedge (\forall V4x \in A_27a.(p \ (\\
& ap \ V1Q \ V4x))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A_27a}).(((p \ V0P) \wedge (\forall V2x \in A_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in \\
& A_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A_27a}).((\forall V2x \in A_27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\
& V0P) \vee (\forall V3x \in A_27a.(p \ (ap \ V1Q \ V3x))))))
\end{aligned} \tag{23}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (27)$$

Assume the following.

$$\forall A. \text{A_27a.nonempty } A. \text{A_27a} \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1a \in A. \text{A_27a}. ((\exists V2x \in A. \text{A_27a}. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A. \text{A_27a.nonempty } A. \text{A_27a} \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A-27a}). \\ & \quad (\forall V1a \in (ty_2Epair_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))). ((\\ & \quad p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A-27a})) V0f) \\ & \quad (ap (ap (c_2Emeasure_2Emeasurable A_27a ty_2Eextreal_2Eextreal) \\ & \quad V1a) c_2Emeasure_2EBorel))) \Leftrightarrow ((p (ap (c_2Emeasure_2Esigma_algebra \\ & \quad A_27a) V1a)) \wedge ((p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A-27a})) \\ & \quad V0f) (ap (ap (c_2Epred_set_2EFUNSET A_27a ty_2Eextreal_2Eextreal) \\ & \quad (ap (c_2Emeasure_2Espace A_27a) V1a)) (c_2Epred_set_2EUNIV \\ & \quad ty_2Eextreal_2Eextreal)))) \wedge (\forall V2c \in ty_2Eextreal_2Eextreal. \\ & \quad (p (ap (ap (c_2Ebool_2EIN (2^{A-27a})) (ap (ap (c_2Epred_set_2EINTER \\ & \quad A_27a) (ap (c_2Epred_set_2EGSPEC A_27a A_27a) (\lambda V3x \in A. \text{A_27a}. \\ & \quad (ap (ap (c_2Epair_2E_2C A_27a 2) V3x) (ap (ap c_2Eextreal_2Eextreal_lt \\ & \quad (ap V0f V3x)) V2c)))))) (ap (c_2Emeasure_2Espace A_27a) V1a)) (\\ & \quad ap (c_2Emeasure_2Esubsets A_27a) V1a)))))) \quad (29) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A-27a}). \\
& \quad (\forall V1g \in (ty_2Eextreal_2Eextreal^{A-27a}). (\forall V2a \in (\\
& \quad ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})})). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& (ty_2Eextreal_2Eextreal^{A-27a})\ V0f)\ (ap\ (ap\ (c_2Emeasure_2E measurable \\
& \quad A_27a\ ty_2Eextreal_2Eextreal)\ V2a)\ c_2Emeasure_2EBorel)))) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A-27a})\ V1g) \\
& \quad (ap\ (ap\ (c_2Emeasure_2E measurable\ A_27a\ ty_2Eextreal_2Eextreal) \\
& \quad V2a)\ c_2Emeasure_2EBorel)))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A-27a}) \\
& (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad A_27a\ A_27a)\ (\lambda V3x \in A_27a.(ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ 2) \\
& V3x)\ (ap\ (ap\ c_2Eextreal_2Eextreal_lt\ (ap\ V0f\ V3x))\ (ap\ V1g\ V3x)))))) \\
& (ap\ (c_2Emeasure_2Espace\ A_27a)\ V2a)))\ (ap\ (c_2Emeasure_2Esubsets \\
& \quad A_27a)\ V2a))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& \quad A_27b. ((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in \\
& (2^{A-27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A-27b}). (\forall V1v \in \\
& A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in \\
& (2^{A-27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V2x)\ V1t))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in \\ & (2^{A-27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (\\ & ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A.27a)\ V2x)\ V1t)))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (50)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\ & \quad (\forall V1g \in (ty_2Eextreal_2Eextreal^{A.27a}). (\forall V2a \in (\\ & \quad ty_2Epair_2Eprod \ (2^{A.27a}) \ (2^{(2^{A.27a})})). (((p \ (ap \ (ap \ (c_2Ebool_2EIN \\ & \quad (ty_2Eextreal_2Eextreal^{A.27a})) \ V0f) \ (ap \ (ap \ (c_2Emeasure_2Emeasurable \\ & \quad A.27a \ ty_2Eextreal_2Eextreal) \ V2a) \ c_2Emeasure_2EBorel)))) \wedge \\ & \quad (p \ (ap \ (ap \ (c_2Ebool_2EIN \ (ty_2Eextreal_2Eextreal^{A.27a})) \ V1g) \\ & \quad (ap \ (ap \ (c_2Emeasure_2Emeasurable \ A.27a \ ty_2Eextreal_2Eextreal) \\ & \quad V2a) \ c_2Emeasure_2EBorel)))) \Rightarrow (p \ (ap \ (ap \ (c_2Ebool_2EIN \ (2^{A.27a})) \\ & \quad (ap \ (ap \ (c_2Epred_set_2EINTER \ A.27a) \ (ap \ (c_2Epred_set_2EGSPEC \\ & \quad A.27a \ A.27a) \ (\lambda V3x \in A.27a. (ap \ (ap \ (c_2Epair_2E_2C \ A.27a \ 2) \\ & \quad V3x) \ (ap \ (ap \ c_2Eextreal_2Eextreal_le \ (ap \ V0f \ V3x)) \ (ap \ V1g \ V3x)))))) \\ & \quad (ap \ (c_2Emeasure_2Espace \ A.27a) \ V2a))) \ (ap \ (c_2Emeasure_2Esubsets \\ & \quad A.27a) \ V2a)))))) \end{aligned}$$