

thm\_2Emeasure\_2EIN\_\_MEASURABLE\_\_BOREL\_\_LT  
 (TM-  
 PLg6a3BVqMGwF4naH5VXTwNNHuVjyeK6A)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a})).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 7** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{2}$$

Let  $c\_2Eextreal\_2ENormal : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \tag{3}$$

Let  $c\_2Eextreal\_2EPosInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \tag{4}$$

Let  $c\_2Eextreal\_2ENegInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \tag{5}$$

**Definition 8** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

**Definition 9** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ ($

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{6}$$

**Definition 11** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2ET)$ .

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27b}$

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 15** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2E\_3F$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{7}$$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in (2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))} \tag{8}$$

**Definition 16** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{9}$$

**Definition 17** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \tag{10}$$

**Definition 18** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (11)$$

**Definition 19** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 20** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$ .

**Definition 21** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E7E)\ s)$ .

**Definition 22** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 23** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$ .

**Definition 24** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 25** We define  $c\_2Eextreal\_2Eextreal\_of\_num$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Ereal\_2Ereal\_of\_num)$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (13)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (14)$$

**Definition 26** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Eextreal\_2Eextreal\_le : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (15)$$

**Definition 27** We define  $c\_2Eextreal\_2Eextreal\_lt$  to be  $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eextreal\_2Eextreal$ .

Let  $c\_2Eextreal\_2Eextreal\_inv : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_inv \in (ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal}) \quad (16)$$

Let  $c\_2Eextreal\_2Eextreal\_mul : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_mul \in ((ty\_2Eextreal\_2Eextreal)^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal} \quad (17)$$

**Definition 28** We define  $c\_2Eextreal\_2Eextreal\_div$  to be  $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eextreal\_2Eextreal$ .

Let  $c\_2Eextreal\_2Eextreal\_ainv : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_ainv \in (ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal}) \quad (18)$$

**Definition 29** We define  $c\_2Eextreal\_2EQ\_set$  to be  $(ap (ap (c\_2Epred\_set\_2EUNION ty\_2Eextreal\_2Eextreal$

**Definition 30** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap (c\_2Epred\_set$

**Definition 31** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod (2^{A-27a})$

**Definition 32** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A-27a}). \lambda V1s \in$

**Definition 33** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap (c\_2Epred\_set$

**Definition 34** We define  $c\_2Emeasure\_2Esigma$  to be  $\lambda A\_27a : \iota. \lambda V0sp \in (2^{A-27a}). \lambda V1st \in (2^{(2^{A-27a})}). (ap$

**Definition 35** We define  $c\_2Emeasure\_2EBorel$  to be  $(ap (ap (c\_2Emeasure\_2Esigma ty\_2Eextreal\_2Eextreal$

**Definition 36** We define  $c\_2Epred\_set\_2EPREIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A-27a}). \lambda V$

**Definition 37** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c$

**Definition 38** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0P \in (2^{A-27a}). \lambda V1Q \in ($

**Definition 39** We define  $c\_2Emeasure\_2Emeasurable$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (21)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t) \Leftrightarrow (p V0t)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (24)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\
& (2^{A\_27a}).((\forall V2x \in A\_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\
& ((\forall V3x \in A\_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A\_27a.(p ( \\
& ap V1Q V4x))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\
& 2.(((\forall V2x \in A\_27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in \\
& A\_27a.((p (ap V0P V3x)) \wedge (p V1Q))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\
& 2^{A\_27a}).(((p V0P) \wedge (\forall V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in \\
& A\_27a.((p V0P) \wedge (p (ap V1Q V3x))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (36)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1a \in A.27a. ((\exists V2x \in A.27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (38)$$

Assume the following.

$$(\forall V0x \in ty.2Eextreal.2Eextreal. ((V0x = c.2Eextreal.2ENegInf) \vee ((V0x = c.2Eextreal.2EPosInf) \vee (\exists V1r \in ty.2Ereal.x.2Ereal. (V0x = (ap c.2Eextreal.2ENormal V1r)))))) \quad (39)$$

Assume the following.

$$(\forall V0x \in ty.2Eextreal.2Eextreal. (\forall V1y \in ty.2Eextreal.2Eextreal. (\forall V2z \in ty.2Eextreal.2Eextreal. (((p (ap (ap c.2Eextreal.2Eextreal._lt V0x) V1y)) \wedge (p (ap (ap c.2Eextreal.2Eextreal._lt V1y) V2z))) \Rightarrow (p (ap (ap c.2Eextreal.2Eextreal._lt V0x) V2z)))))) \quad (40)$$

Assume the following.

$$(\forall V0r \in ty.2Eextreal.2Eextreal. ((p (ap (ap (c.2Ebool.2EIN ty.2Eextreal.2Eextreal) V0r) c.2Eextreal.2EQ._set)) \Rightarrow ((\neg (V0r = c.2Eextreal.2ENegInf)) \wedge (\neg (V0r = c.2Eextreal.2EPosInf)))))) \quad (41)$$

Assume the following.

$$(\forall V0x \in ty.2Eextreal.2Eextreal. (\forall V1y \in ty.2Eextreal.2Eextreal. ((p (ap (ap c.2Eextreal.2Eextreal._lt V0x) V1y)) \Rightarrow (\exists V2r \in ty.2Eextreal.2Eextreal. ((p (ap (ap (c.2Ebool.2EIN ty.2Eextreal.2Eextreal) V2r) c.2Eextreal.2EQ._set)) \wedge ((p (ap (ap c.2Eextreal.2Eextreal._lt V0x) V2r)) \wedge (p (ap (ap c.2Eextreal.2Eextreal._lt V2r) V1y)))))) \quad (42)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A_{.27a}}) (2^{(2^{A_{.27a}})})), (\forall V1s \in (2^{A_{.27a}}), (\forall V2t \in \\
& \quad (2^{A_{.27a}}), (((p (ap (c\_2Emeasure\_2Ealgebra\ A_{.27a})\ V0a)) \wedge ((p ( \\
& \quad ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}}))\ V1s) (ap (c\_2Emeasure\_2Esubsets \\
& \quad A_{.27a})\ V0a))) \wedge (p (ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}}))\ V2t) (ap (c\_2Emeasure\_2Esubsets \\
& \quad A_{.27a})\ V0a)))))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}})) (ap (ap (c\_2Epred\_set\_2EINTER \\
& \quad A_{.27a})\ V1s)\ V2t)) (ap (c\_2Emeasure\_2Esubsets\ A_{.27a})\ V0a))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A_{.27a}}) (2^{(2^{A_{.27a}})})), (\forall V1f \in ((2^{A_{.27a}}) ty\_2Eextreal\_2Eextreal)), \\
& \quad (((p (ap (c\_2Emeasure\_2Esigma\_algebra\ A_{.27a})\ V0a)) \wedge (p (ap (ap \\
& \quad (c\_2Ebool\_2EIN ((2^{A_{.27a}}) ty\_2Eextreal\_2Eextreal))\ V1f) (ap \\
& \quad (ap (c\_2Epred\_set\_2EFUNSET\ ty\_2Eextreal\_2Eextreal (2^{A_{.27a}})) \\
& \quad c\_2Eextreal\_2EQ\_set) (ap (c\_2Emeasure\_2Esubsets\ A_{.27a})\ V0a)))))) \Rightarrow \\
& \quad (p (ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}})) (ap (c\_2Epred\_set\_2EBIGUNION \\
& \quad A_{.27a}) (ap (ap (c\_2Epred\_set\_2EIMAGE\ ty\_2Eextreal\_2Eextreal \\
& \quad (2^{A_{.27a}}))\ V1f)\ c\_2Eextreal\_2EQ\_set))) (ap (c\_2Emeasure\_2Esubsets \\
& \quad A_{.27a})\ V0a))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A_{.27a}}), \\
& \quad (\forall V1a \in (ty\_2Epair\_2Eprod (2^{A_{.27a}}) (2^{(2^{A_{.27a}})})), ((( \\
& \quad p (ap (ap (c\_2Ebool\_2EIN (ty\_2Eextreal\_2Eextreal^{A_{.27a}}))\ V0f) \\
& \quad (ap (ap (c\_2Emeasure\_2Emeasurable\ A_{.27a}\ ty\_2Eextreal\_2Eextreal) \\
& \quad V1a)\ c\_2Emeasure\_2EBorel))) \Leftrightarrow ((p (ap (c\_2Emeasure\_2Esigma\_algebra \\
& \quad A_{.27a})\ V1a)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (ty\_2Eextreal\_2Eextreal^{A_{.27a}})) \\
& \quad V0f) (ap (ap (c\_2Epred\_set\_2EFUNSET\ A_{.27a}\ ty\_2Eextreal\_2Eextreal) \\
& \quad (ap (c\_2Emeasure\_2Espace\ A_{.27a})\ V1a)) (c\_2Epred\_set\_2EUNIV \\
& \quad ty\_2Eextreal\_2Eextreal)))) \wedge (\forall V2c \in ty\_2Eextreal\_2Eextreal, \\
& \quad (p (ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}})) (ap (ap (c\_2Epred\_set\_2EINTER \\
& \quad A_{.27a}) (ap (c\_2Epred\_set\_2EGSPEC\ A_{.27a}\ A_{.27a}) (\lambda V3x \in A_{.27a}. \\
& \quad (ap (ap (c\_2Epair\_2E.2C\ A_{.27a}\ 2)\ V3x) (ap (ap c\_2Eextreal\_2Eextreal\_lt \\
& \quad (ap\ V0f\ V3x))\ V2c)))))) (ap (c\_2Emeasure\_2Espace\ A_{.27a})\ V1a))) ( \\
& \quad ap (c\_2Emeasure\_2Esubsets\ A_{.27a})\ V1a))))))
\end{aligned} \tag{45}$$





Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in \\ & \quad A\_27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & \quad (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \\ & \end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}). (\forall V1v \in \\ & \quad A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & \quad A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \\ & \end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & \quad (2^{A\_27a}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ \\ & \quad V0s)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & \quad (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27a)\ V2x)\ V1t)))))) \\ & \end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}). (\forall V1P \in (2^{A\_27a}). (\forall V2Q \in \\ & \quad (2^{A\_27b}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A\_27b^{A\_27a}))\ V0f)\ (ap\ (ap \\ & \quad (c\_2Epred\_set\_2EFUNSET\ A\_27a\ A\_27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\ & \quad A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27b)\ (ap\ V0f\ V3x))\ V2Q)))))) \\ & \end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in ((2^{A\_27b})^{A\_27a}). (\forall V1s \in (2^{A\_27a}). (\forall V2y \in \\ & \quad A\_27b. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V2y)\ (ap\ (c\_2Epred\_set\_2EBIGUNION \\ & \quad A\_27b)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27a\ (2^{A\_27b}))\ V0f)\ V1s)))) \Leftrightarrow \\ & \quad (\exists V3x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s)) \wedge \\ & \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V2y)\ (ap\ V0f\ V3x)))))) \\ & \end{aligned} \tag{52}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (67)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A-27a}). \\ & \quad (\forall V1g \in (ty\_2Eextreal\_2Eextreal^{A-27a}).(\forall V2a \in ( \\ & \quad ty\_2Epair\_2Eprod \ (2^{A-27a}) \ (2^{(2^{A-27a})})).(((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \\ & (ty\_2Eextreal\_2Eextreal^{A-27a})) \ V0f) \ (ap \ (ap \ (c\_2Emeasure\_2E measurable \\ & \ A.27a \ ty\_2Eextreal\_2Eextreal) \ V2a) \ c\_2Emeasure\_2EBorel)))) \wedge \\ & \quad (p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ (ty\_2Eextreal\_2Eextreal^{A-27a})) \ V1g) \\ & \quad (ap \ (ap \ (c\_2Emeasure\_2E measurable \ A.27a \ ty\_2Eextreal\_2Eextreal) \\ & \quad V2a) \ c\_2Emeasure\_2EBorel)))) \Rightarrow (p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ (2^{A-27a}) \\ & \quad (ap \ (ap \ (c\_2Epred\_set\_2EINTER \ A.27a) \ (ap \ (c\_2Epred\_set\_2EGSPEC \\ & \ A.27a \ A.27a) \ (\lambda V3x \in A.27a.(ap \ (ap \ (c\_2Epair\_2E.2C \ A.27a \ 2) \\ & \ V3x) \ (ap \ (ap \ c\_2Eextreal\_2Eextreal\_lt \ (ap \ V0f \ V3x)) \ (ap \ V1g \ V3x)))))) \\ & \quad (ap \ (c\_2Emeasure\_2Espace \ A.27a) \ V2a))) \ (ap \ (c\_2Emeasure\_2Esubsets \\ & \ A.27a) \ V2a)))))) \end{aligned}$$