thm_2Emeasure_2EIN__MEASURABLE__BOREL__MONO__SUP (TMYt9Wzv5Siyry6Yo9aMh1i3Z3PsT81TE4u)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 5 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 6 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27c}$

Definition 7 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 8 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A\_$

Definition 9 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \quad (5)$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (6)$$

Let $c\_2Eextreal\_2ENormal : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \quad (7)$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (8)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (9)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \quad (10)$$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (11)$$

Definition 12 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 13 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 16 *We define* c_2Ereal_2Esup *to be* $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Emin_2E_40 ty_2Ereal

Let $c\_2Eextreal\_2ENegInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \quad (12)$$

Definition 17 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Eextreal\_2EPosInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \tag{13}$$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{14}$$

Definition 19 *We define* c_2Eextreal_2Eextreal__sup *to be* $\lambda V0p \in (2^{ty\_2Eextreal\_2Eextreal}).(ap\ (ap\ (ap\ (\mathsf{c\_2E}$

Definition 20 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 21 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{15}$$

Definition 22 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{16}$$

Definition 23 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 24 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 25 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 26 *We define* c_2Emeasure_2Esubset__class *to be* $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27}}$

Let $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \tag{17}$$

Let $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \tag{18}$$

Definition 27 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 28 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 29 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Definition 30 *We define* c_2Epred__set_2Ecountable *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_3F

Definition 31 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 32 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 33 *We define* c_2Emeasure_2Ealgebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2}$

Definition 34 *We define* c_2Emeasure_2Esigma__algebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_2}$

Definition 35 *We define* c_2Eextreal_2Eextreal__lt *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eext$

Definition 36 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 37 *We define* c_2Epred__set_2EBIGINTER *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 38 *We define* c_2Emeasure_2Esigma *to be* $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1st \in (2^{(2^{A\_27a})}).(ap\ ($

Definition 39 *We define* c_2Emeasure_2EBorel *to be* $(ap\ (ap$ (c_2Emeasure_2Esigma $ty\_2Eextreal\_2Eextre$

Definition 40 *We define* c_2Epred__set_2EPREIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V$

Definition 41 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 42 *We define* c_2Epred__set_2EFUNSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in ($

Definition 43 *We define* c_2Emeasure_2Emeasurable *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (ty\_2Epair\_2Epro$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t))\Rightarrow((p\ V0t)\Rightarrow False))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{25}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(V0x = V0x)) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{30}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{31}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27b)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)) = V0f))) \tag{35}$$

Assume the following.

$$(\forall V0p \in (2^{ty\_2Eextreal\_2Eextreal}).(\forall V1x \in ty\_2Eextreal\_2Eextreal.((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_sup}\ V0p))\ V1x)) \Leftrightarrow (\forall V2y \in ty\_2Eextreal\_2Eextreal.((p\ (ap\ V0p\ V2y)) \Rightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V2y)\ V1x))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V0x))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V1y))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0a))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a)) = V0a)) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))).((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V0a)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Esubset\_\_class}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0a))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a))) \wedge ((\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V1s)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0a))\ V1s))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a))))) \wedge (\forall V2f \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ((2^{A\_27a})^{ty\_2Enum\_2Enum}))\ V2f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ ty\_2Enum\_2Enum\ (2^{A\_27a}))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Enum\_2Enum))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGINTER}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Enum\_2Enum\ (2^{A\_27a}))\ V2f)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Enum\_2Enum))))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a))))))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V1a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eextreal\_2Eextreal^{A\_27a}))\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ ty\_2Eextreal\_2Eextreal)\ V1a)\ \mathsf{c\_2Emeasure\_2EBorel}))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V1a)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eextreal\_2Eextreal^{A\_27a}))\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ ty\_2Eextreal\_2Eextreal)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V1a))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Eextreal\_2Eextreal)))) \wedge (\forall V2c \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V3x)\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ V0f\ V3x))\ V2c)))))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V1a)))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V1a))))))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0P)) \Leftrightarrow (p\ (ap\ V0P\ V1x))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V0y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s)))))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27a}).(\forall V2Q \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ A\_27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0f\ V3x))\ V2Q)))))))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in ((2^{A\_27a})^{A\_27b}).(\forall V2s \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGINTER}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27b\ (2^{A\_27a}))\ V1f)\ V2s)))) \Leftrightarrow (\forall V3y \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V3y)\ V2s)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ V1f\ V3y))))))))) \tag{50}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor ((p\ V1q) \lor (p\ V2r))) \land (((p\ V0p) \lor ((\neg(p\ V2r)) \lor (\neg(p\ V1q)))) \land (((p\ V1q) \lor ((\neg(p\ V2r)) \lor (\neg(p\ V0p)))) \land ((p\ V2r) \lor ((\neg(p\ V1q)) \lor (\neg(p\ V0p))))))))))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \lor (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor (\neg(p\ V1q))) \land (((p\ V0p) \lor (\neg(p\ V2r))) \land ((p\ V1q) \lor ((p\ V2r) \lor (\neg(p\ V0p)))))))))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor (p\ V1q)) \land (((p\ V0p) \lor (\neg(p\ V2r))) \land ((\neg(p\ V1q)) \lor ((p\ V2r) \lor (\neg(p\ V0p)))))))))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{64}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0fn \in ((ty\_2Eextreal\_2Eextreal^{A\_27a})^{ty\_2Enum\_2Enum}). (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V2a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).(((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V2a))\wedge((\forall V3n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eextreal\_2Eextreal^{A\_27a}))\ (ap\ V0fn\ V3n))\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ ty\_2Eextreal\_2Eextreal)\ V2a)\ \mathsf{c\_2Emeasure\_2EBorel}))))\wedge ((\forall V4n \in ty\_2Enum\_2Enum.(\forall V5x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5x)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V2a)))\Rightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ V0fn\ V4n)\ V5x))\ (ap\ (ap\ V0fn\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V4n))\ V5x))))))\wedge(\forall V6x \in A\_27a. ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V6x)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V2a)))\Rightarrow((ap\ V1f\ V6x) = (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_sup}\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Enum\_2Enum\ ty\_2Eextreal\_2Eextreal)\ (\lambda V7n \in ty\_2Enum\_2Enum.(ap\ (ap\ V0fn\ V7n)\ V6x)))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Enum\_2Enum)))))))))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eextreal\_2Eextreal^{A\_27a}))\ V1f)\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ ty\_2Eextreal\_2Eextreal)\ V2a)\ \mathsf{c\_2Emeasure\_2EBorel}))))))$$