

thm_2Emeasure_2EIN__MEASURABLE__BOREL__MUL__INDICA
(TMWK8wmqPVC05PuJXyVVkBLdBg6fzsJB9jV)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 5 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 6 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P) P))$

Definition 8 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27a}).$

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 P) P)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(3)

Definition 14 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})})$.

Definition 15 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a})$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum$$
(4)

Definition 16 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 17 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b})$.

Definition 18 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a})$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(5)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(6)

Definition 19 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 20 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
(7)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
(8)

Definition 21 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$
(9)

Definition 22 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 23 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (10)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (12)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (13)$$

Definition 24 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in (\quad (14)$$

$$(2^{(2^{A-27a})})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))})$$

Definition 25 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in$

Definition 26 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c_2$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))}) \quad (15)$$

Definition 27 We define c_2Ebool_2E5F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 28 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 29 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c_2$

Definition 30 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E5F)$.

Definition 31 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1sts \in (2^{(2^{A-27a})})$

Definition 32 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((ty_2Eextreal_2Eextreal)^{ty_2Eextreal_2Eextreal}) \quad (16)$$

Definition 33 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eext$

Definition 34 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 35 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (\lambda V1x \in A_27a. (ap$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})ty_2Eextreal_2Eextreal) \quad (17)$$

Definition 36 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in$

Definition 37 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}$

Definition 38 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap (c_2Epred_set$

Definition 39 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A_27a}). \lambda V1st \in (2^{(2^{A_27a})}). (ap$

Definition 40 We define $c_2Emeasure_2EBorel$ to be $(ap (ap (c_2Emeasure_2Esigma ty_2Eextreal_2Eextreal$

Definition 41 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V$

Definition 42 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c$

Definition 43 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0P \in (2^{A_27a}). \lambda V1Q \in ($

Definition 44 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0a \in (ty_2Epair_2Eprod$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (20)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (32)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0b \in 2. (\forall V1f \in (A_27b^{A_27a}). (\forall V2g \in (A_27b^{A_27a}). \\
& \quad (\forall V3x \in A_27a. ((ap\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (A_27b^{A_27a}) \\
V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V0b)\ (ap \\
& \quad \quad \quad V1f\ V3x))\ (ap\ V2g\ V3x))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\
& \quad (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
V1b)\ V2x)\ V3y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f \\
& \quad \quad \quad V2x))\ (ap\ V0f\ V3y))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\
& \quad 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \tag{35} \\
& \quad (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\
& \quad A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (\\
& \quad \quad \quad ap\ V0P\ V1a))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI \\
& \quad \quad \quad A_27a)\ V0x) = V0x))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b \\
A_27b)\ (c_2Ecombin_2EI\ A_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo \\
& \quad \quad \quad A_27a\ A_27b\ A_27a)\ V0f)\ (c_2Ecombin_2EI\ A_27a)) = V0f)))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. ((ap\ (ap\ c_2Eextreal_2Eextreal_mul \\
V0x)\ (ap\ c_2Eextreal_2Eextreal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad \quad \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) = V0x))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\ & (2^{A.27a}) (2^{(2^{A.27a})})).((p (ap (c_2Emeasure_2Ealgebra\ A.27a) \\ & V0a)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN (2^{A.27a})) (ap (c_2Emeasure_2Espace \\ & A.27a) V0a)) (ap (c_2Emeasure_2Esubsets\ A.27a) V0a)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\ & (\forall V1a \in (ty_2Epair_2Eprod (2^{A.27a}) (2^{(2^{A.27a})})).((\\ & p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A.27a})) V0f) \\ & (ap (ap (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\ & V1a) c_2Emeasure_2EBorel))) \Leftrightarrow ((p (ap (c_2Emeasure_2Esigma_algebra \\ & A.27a) V1a)) \wedge ((p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A.27a})) \\ & V0f) (ap (ap (c_2Epred_set_2EFUNSET\ A.27a\ ty_2Eextreal_2Eextreal) \\ & (ap (c_2Emeasure_2Espace\ A.27a) V1a)) (c_2Epred_set_2EUNIV \\ & ty_2Eextreal_2Eextreal)))) \wedge (\forall V2c \in ty_2Eextreal_2Eextreal. \\ & (p (ap (ap (c_2Ebool_2EIN (2^{A.27a})) (ap (ap (c_2Epred_set_2EINTER \\ & A.27a) (ap (c_2Epred_set_2EGSPEC\ A.27a\ A.27a) (\lambda V3x \in A.27a. \\ & (ap (ap (c_2Epair_2E_2C\ A.27a\ 2) V3x) (ap (ap c_2Eextreal_2Eextreal_lt \\ & (ap V0f V3x)) V2c)))))) (ap (c_2Emeasure_2Espace\ A.27a) V1a)) (\\ & ap (c_2Emeasure_2Esubsets\ A.27a) V1a))))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\ & (2^{A.27a}) (2^{(2^{A.27a})})).(\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\ & (\forall V2s \in (2^{A.27a}).(((p (ap (c_2Emeasure_2Esigma_algebra \\ & A.27a) V0a)) \wedge ((p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A.27a})) \\ & V1f) (ap (ap (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\ & V0a) c_2Emeasure_2EBorel))) \wedge (p (ap (ap (c_2Ebool_2EIN (2^{A.27a}) \\ & V2s) (ap (c_2Emeasure_2Esubsets\ A.27a) V0a)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN \\ & (ty_2Eextreal_2Eextreal^{A.27a})) (\lambda V3x \in A.27a.(ap (ap c_2Eextreal_2Eextreal_mul \\ & (ap V1f V3x)) (ap (ap (c_2Emeasure_2Eindicator_fn\ A.27a) V2s) \\ & V3x)))) (ap (ap (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\ & V0a) c_2Emeasure_2EBorel))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\ & A.27b.(((ap (ap (c_2Epair_2E_2C\ A.27a\ A.27b) V0x) V1y) = (ap (ap \\ & (c_2Epair_2E_2C\ A.27a\ A.27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A_27a)))) \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27a}). (\forall V2Q \in \\ & (2^{A_27b}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A_27b^{A_27a})\ V0f)\ (ap\ (ap \\ & (c_2Epred_set_2EFUNSET\ A_27a\ A_27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\ & A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ (ap\ V0f\ V3x)\ V2Q)))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(\forall V3s \in 2.(((p V0p) \Leftrightarrow (p (ap (ap (ap (c_2Ebool_2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V3s)))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p)))))))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg(p V0p) \Rightarrow (p V1q)) \Rightarrow (p V0p)))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (64)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\ & \quad (2^{A.27a}) \ (2^{(2^{A.27a})})).(\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\ & \quad ((p \ (ap \ (c_2Emeasure_2Esigma_algebra \ A.27a) \ V0a)) \Rightarrow ((p \ (ap \ (ap \\ & \quad \quad (c_2Ebool_2EIN \ (ty_2Eextreal_2Eextreal^{A.27a}) \ V1f) \ (ap \ (ap \ (\\ & \quad \quad \quad c_2Emeasure_2Emeasurable \ A.27a \ ty_2Eextreal_2Eextreal) \ V0a) \\ & \quad \quad c_2Emeasure_2EBorel))) \Leftrightarrow (p \ (ap \ (ap \ (c_2Ebool_2EIN \ (ty_2Eextreal_2Eextreal^{A.27a}) \\ & \quad \quad (\lambda V2x \in A.27a.(ap \ (ap \ c_2Eextreal_2Eextreal_mul \ (ap \ V1f \ V2x)) \\ & \quad \quad (ap \ (ap \ (c_2Emeasure_2Eindicator_fn \ A.27a) \ (ap \ (c_2Emeasure_2Espace \\ & \quad \quad A.27a) \ V0a)) \ V2x)))) \ (ap \ (ap \ (c_2Emeasure_2Emeasurable \ A.27a \ ty_2Eextreal_2Eextreal \\ & \quad \quad \quad V0a) \ c_2Emeasure_2EBorel))))))))) \end{aligned}$$