thm_2Emeasure_2ELAMBDA__SYSTEM__ADDITIVE (TMH-WNKFj1vreUwDMSEdcj5bpESthrhYHPbj)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})} \tag{3}$$

Let $c\_2Emeasure\_2Emeasurable\_\_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \tag{4}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{5}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{6}$$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{7}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{8}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{9}$$

Definition 9 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 10 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 11 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 13 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{10}$$

Definition 14 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{11}$$

Definition 15 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 16 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 17 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 18 *We define* c_2Epred__set_2EDISJOINT *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

Definition 19 *We define* c_2Emeasure_2Eadditive *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (t$

Definition 20 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Let $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (12)$$

Let $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (13)$$

Definition 21 *We define* c_2Emeasure_2Elambda__system *to be* $\lambda A\_27a : \iota.\lambda V0gen \in (ty\_2Epair\_2Eprod\ (2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (14)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (15)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (16)$$

Definition 22 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (17)$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \quad (18)$$

Definition 23 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 24 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 25 *We define* c_2Emeasure_2Epositive *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (t$

Definition 26 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 27 *We define* c_2Emeasure_2Esubset__class *to be* $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27}}$

Definition 28 *We define* c_2Emeasure_2Ealgebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2}$

Definition 29 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Assume the following.
$$True \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{27}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a)\wedge(p\ (ap\ V0P\ V2x))))\Leftrightarrow(p\ (ap\ V0P\ V1a))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0sp \in (2^{A\_27a}).(\forall V1sts \in (2^{(2^{A\_27a})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). ((ap\ (c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))) V0sp)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) V1sts)\ V2mu))) = V1sts)))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0sp \in (2^{A\_27a}).(\forall V1sts \in (2^{(2^{A\_27a})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). ((ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a}) (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))) V0sp)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) V1sts)\ V2mu))) = V2mu)))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a})\ (2^{(2^{A\_27a})})).((p\ (ap\ (\mathsf{c\_2Emeasure\_2Ealgebra}\ A\_27a) V0a))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (c\_2Emeasure\_2Espace A\_27a)\ V0a))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a))))) \tag{32}$$

Assume the following.

$$\forall \mathit{A\_27a}.\mathit{nonempty}\ \mathit{A\_27a}\Rightarrow(\forall \mathit{V0a} \in (\mathit{ty\_2Epair\_2Eprod}\ (2^{\mathit{A\_27a}})\ (2^{(2^{\mathit{A\_27a}})})).(\forall \mathit{V1s} \in (2^{\mathit{A\_27a}}).(\forall \mathit{V2t} \in (2^{\mathit{A\_27a}}).(((p\ (\mathit{ap}\ (\mathsf{c\_2Emeasure\_2Ealgebra}\ \mathit{A\_27a})\ \mathit{V0a}))\wedge((p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{\mathit{A\_27a}}))\ \mathit{V1s})\ (\mathit{ap}\ (\mathit{c\_2Emeasure\_2Esubsets}\ \mathit{A\_27a})\ \mathit{V0a})))\wedge(p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{\mathit{A\_27a}}))\ \mathit{V2t})\ (\mathit{ap}\ (\mathit{c\_2Emeasure\_2Esubsets}\ \mathit{A\_27a})\ \mathit{V0a})))))\Rightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{\mathit{A\_27a}}))\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ \mathit{A\_27a})\ \mathit{V1s})\ \mathit{V2t}))\ (\mathit{ap}\ (\mathit{c\_2Emeasure\_2Esubsets}\ \mathit{A\_27a})\ \mathit{V0a}))))))) \tag{33}$$

Assume the following.

$$\forall \mathit{A\_27a}.\mathit{nonempty}\ \mathit{A\_27a}\Rightarrow(\forall \mathit{V0a} \in (\mathit{ty\_2Epair\_2Eprod}\ (2^{\mathit{A\_27a}})\ (2^{(2^{\mathit{A\_27a}})})).(\forall \mathit{V1s} \in (2^{\mathit{A\_27a}}).(((p\ (\mathit{ap}\ (\mathsf{c\_2Emeasure\_2Ealgebra}\ \mathit{A\_27a})\ \mathit{V0a}))\wedge(p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{\mathit{A\_27a}}))\ \mathit{V1s})\ (\mathit{ap}\ (\mathit{c\_2Emeasure\_2Esubsets}\ \mathit{A\_27a})\ \mathit{V0a}))))\Rightarrow(((\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ \mathit{A\_27a})\ (\mathit{ap}\ (\mathit{c\_2Emeasure\_2Espace}\ \mathit{A\_27a})\ \mathit{V0a}))\ \mathit{V1s}) = \mathit{V1s})\wedge((\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ \mathit{A\_27a})\ \mathit{V1s})\ (\mathit{ap}\ (\mathit{c\_2Emeasure\_2Espace}\ \mathit{A\_27a})\ \mathit{V0a})) = \mathit{V1s}))))) \tag{34}$$

Assume the following.

$$\forall \mathit{A\_27a}.\mathit{nonempty}\ \mathit{A\_27a}\Rightarrow(\forall \mathit{V0g0} \in (\mathit{ty\_2Epair\_2Eprod}\ (2^{\mathit{A\_27a}})\ (2^{(2^{\mathit{A\_27a}})})).(\forall \mathit{V1lam} \in (\mathit{ty\_2Erealax\_2Ereal}^{(2^{\mathit{A\_27a}})}).(\forall \mathit{V2g} \in (2^{\mathit{A\_27a}}).(\forall \mathit{V3l1} \in (2^{\mathit{A\_27a}}).(\forall \mathit{V4l2} \in (2^{\mathit{A\_27a}}).(((p\ (\mathit{ap}\ (\mathsf{c\_2Emeasure\_2Ealgebra}\ \mathit{A\_27a})\ \mathit{V0g0}))\wedge((p\ (\mathit{ap}\ (\mathsf{c\_2Emeasure\_2Epositive}\ \mathit{A\_27a})\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{\mathit{A\_27a}})\ (\mathit{ty\_2Epair\_2Eprod}\ (2^{(2^{\mathit{A\_27a}})})\ (\mathit{ty\_2Erealax\_2Ereal}^{(2^{\mathit{A\_27a}})})))\ (\mathit{ap}\ (\mathit{c\_2Emeasure\_2Espace}\ \mathit{A\_27a})\ \mathit{V0g0}))\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{\mathit{A\_27a}})})\ (\mathit{ty\_2Erealax\_2Ereal}^{(2^{\mathit{A\_27a}})}))\ (\mathit{ap}\ (\mathit{c\_2Emeasure\_2Esubsets}\ \mathit{A\_27a})\ \mathit{V0g0}))\ \mathit{V1lam}))))\wedge((p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{\mathit{A\_27a}}))\ \mathit{V2g})\ (\mathit{ap}\ (\mathit{c\_2Emeasure\_2Esubsets}\ \mathit{A\_27a})\ \mathit{V0g0})))\wedge((p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EDISJOINT}\ \mathit{A\_27a})\ \mathit{V3l1})\ \mathit{V4l2}))\wedge((p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{\mathit{A\_27a}}))\ \mathit{V3l1})\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Emeasure\_2Elambda\_\_system}\ \mathit{A\_27a})\ \mathit{V0g0})\ \mathit{V1lam})))\wedge(p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{\mathit{A\_27a}}))\ \mathit{V4l2})\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Emeasure\_2Elambda\_\_system}\ \mathit{A\_27a})\ \mathit{V0g0})\ \mathit{V1lam}))))))))\Rightarrow((\mathit{ap}\ \mathit{V1lam}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ \mathit{A\_27a})\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ \mathit{A\_27a})\ \mathit{V3l1})\ \mathit{V4l2}))\ \mathit{V2g})) = (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (\mathit{ap}\ \mathit{V1lam}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ \mathit{A\_27a})\ \mathit{V3l1})\ \mathit{V2g})))\ (\mathit{ap}\ \mathit{V1lam}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ \mathit{A\_27a})\ \mathit{V4l2})\ \mathit{V2g}))))))))) \tag{35}$$

Assume the following.

$$\forall \mathit{A\_27a}.\mathit{nonempty}\ \mathit{A\_27a}\Rightarrow\forall \mathit{A\_27b}.\mathit{nonempty}\ \mathit{A\_27b}\Rightarrow(\forall \mathit{V0x} \in \mathit{A\_27a}.(\forall \mathit{V1y} \in \mathit{A\_27b}.(\forall \mathit{V2a} \in \mathit{A\_27a}.(\forall \mathit{V3b} \in \mathit{A\_27b}.(((\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{A\_27a}\ \mathit{A\_27b})\ \mathit{V0x})\ \mathit{V1y}) = (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{A\_27a}\ \mathit{A\_27b})\ \mathit{V2a})\ \mathit{V3b}))\Leftrightarrow((\mathit{V0x} = \mathit{V2a})\wedge(\mathit{V1y} = \mathit{V3b}))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{37}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0g0 \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})). (\forall V1lam \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}).(\forall V2l1 \in A\_27b.(\forall V3l2 \in A\_27c.(((p\ (ap\ (\mathsf{c\_2Emeasure\_2Ealgebra}\ A\_27a)\ V0g0)) \wedge (p\ (ap\ (\mathsf{c\_2Emeasure\_2Epositive}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0g0))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0g0))\ V1lam))))) \Rightarrow (p\ (ap\ (\mathsf{c\_2Emeasure\_2Eadditive}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0g0))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Elambda\_\_system}\ A\_27a)\ V0g0)\ V1lam))\ V1lam)))))))))$$