

thm_2Emeasure_2ELAMBDA_SYSTEM_ALGEBRA (TMExp4f2YHXCp6y9VrrSeGty1BshZKDJXP3)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_7E$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 7 We define $c_2Ebool_2E_IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Eprel_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) \quad (5)$$

Definition 11 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 12 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ A_27a \in & ((2^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (6)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (7)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (9)$$

Definition 13 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasure A_27a \in (\\ & (ty_2Erealax_2Ereal^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (11)$$

Let $c_2Erealax_2Etrealt_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (12)$$

Let $c_2Erealax_2Etrealt_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (13)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})}) \quad (14)$$

Definition 14 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty$

Definition 15 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 16 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1sts \in (2^{(2^{A-27a})})$

Definition 17 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E2F)$.

Definition 18 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A-27b})^{A-27a}}) \end{aligned} \quad (15)$$

Definition 19 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A-27b}}) \end{aligned} \quad (16)$$

Definition 20 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c_2E$

Definition 21 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c_2E$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))}) \quad (17)$$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in (2^{(2^{A-27a})})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))}) \quad (18)$$

Definition 22 We define $c_2Emeasure_2Elambda_system$ to be $\lambda A_27a : \iota.\lambda V0gen \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))$

Definition 23 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))$

Definition 24 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2$

Definition 25 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c_2E$

Definition 26 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})}))$

Definition 27 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. ((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (27)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}). (\forall V1y \in (2^{(2^{A_27a})}). ((ap\ (c_2Emeasure_2Espace\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})}))\ V0x)\ V1y)) = V0x))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}). (\forall V1y \in (2^{(2^{A_27a})}). ((ap\ (c_2Emeasure_2Esubsets\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})}))\ V0x)\ V1y)) = V1y))) \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})). ((p\ (ap\ (c_2Emeasure_2Ealgebra\ A_27a)\ V0a)) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Emeasure_2Esubset_class\ A_27a)\ (ap\ (c_2Emeasure_2Espace\ A_27a)\ V0a))\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a))) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ (c_2Epred_set_2EEMPTY\ A_27a))\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a))) \wedge ((\forall V1s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V1s)\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_27a)\ (ap\ (c_2Emeasure_2Espace\ A_27a)\ V0a))\ V1s))\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a)))))) \wedge ((\forall V2s \in (2^{A_27a}). (\forall V3t \in (2^{A_27a}). (((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2s)\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a))) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V3t)\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V2s)\ V3t))\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0a))))))))))))) \quad (32) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0g0 \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})). (\forall V1lam \in (ty_2Erealax_2Ereal^{(2^{A_27a})}). (((p\ (ap\ (c_2Emeasure_2Ealgebra\ A_27a)\ V0g0)) \wedge (p\ (ap\ (c_2Emeasure_2Epositive\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))))\ (ap\ (c_2Emeasure_2Espace\ A_27a)\ V0g0))\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))\ (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0g0))\ V1lam)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ (c_2Epred_set_2EEMPTY\ A_27a))\ (ap\ (ap\ (c_2Emeasure_2Elambda_system\ A_27a)\ V0g0)\ V1lam)))))) \quad (33) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g0 \in (ty_2Epair_2Eprod \\
& \quad (2^{A.27a})\ (2^{(2^{A.27a})})).(\forall V1lam \in (ty_2Erealax_2Ereal^{(2^{A.27a})}). \\
& \quad (\forall V2l \in (2^{A.27a}).(((p\ (ap\ (c_2Emeasure_2Ealgebra\ A.27a) \\
& \quad V0g0)) \wedge ((p\ (ap\ (c_2Emeasure_2Epositive\ A.27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))) \\
& \quad (ap\ (c_2Emeasure_2Espace\ A.27a)\ V0g0))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))\ (ap\ (c_2Emeasure_2Esubsets \\
& \quad A.27a)\ V0g0))\ V1lam)))) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a})\ V2l) \\
& \quad (ap\ (ap\ (c_2Emeasure_2Elambda_system\ A.27a)\ V0g0)\ V1lam)))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a})\ (ap\ (ap\ (c_2Epred_set_2EDIFF \\
& \quad A.27a)\ (ap\ (c_2Emeasure_2Espace\ A.27a)\ V0g0))\ V2l))\ (ap\ (ap\ (c_2Emeasure_2Elambda_system \\
& \quad A.27a)\ V0g0)\ V1lam))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g0 \in (ty_2Epair_2Eprod \\
& \quad (2^{A.27a})\ (2^{(2^{A.27a})})).(\forall V1lam \in (ty_2Erealax_2Ereal^{(2^{A.27a})}). \\
& \quad (\forall V2l1 \in (2^{A.27a}).(\forall V3l2 \in (2^{A.27a}).(((p\ (ap\ (c_2Emeasure_2Ealgebra \\
& \quad A.27a)\ V0g0)) \wedge ((p\ (ap\ (c_2Emeasure_2Epositive\ A.27a)\ (ap\ (ap\ (\\
& \quad c_2Epair_2E_2C\ (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))) \\
& \quad (ap\ (c_2Emeasure_2Espace\ A.27a)\ V0g0))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))\ (ap\ (c_2Emeasure_2Esubsets \\
& \quad A.27a)\ V0g0))\ V1lam)))) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a}) \\
& \quad V2l1)\ (ap\ (ap\ (c_2Emeasure_2Elambda_system\ A.27a)\ V0g0)\ V1lam)))) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a})\ V3l2)\ (ap\ (ap\ (c_2Emeasure_2Elambda_system \\
& \quad A.27a)\ V0g0)\ V1lam)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a}) \\
& \quad (ap\ (ap\ (c_2Epred_set_2EINTER\ A.27a)\ V2l1)\ V3l2))\ (ap\ (ap\ (c_2Emeasure_2Elambda_system \\
& \quad A.27a)\ V0g0)\ V1lam))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\
& \quad A.27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Epair_2Eprod\ A.27a\ 2)^{A.27b}).(\forall V1v \in \\
& \quad A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad A.27a\ A.27b)\ V0f))) \Leftrightarrow (\exists V2x \in A.27b.((ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A.27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x))))))
\end{aligned} \tag{37}$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0g0 \in (\text{ty_2Epair_2Eprod} \\ & \quad (2^{A_{27a}}) (2^{(2^{A_{27a}})})) . (\forall V1lam \in (\text{ty_2Erealax_2Ereal}^{(2^{A_{27a}})})) . \\ & \quad (((p (ap (c_2Emeasure_2Ealgebra A_{27a}) V0g0)) \wedge (p (ap (c_2Emeasure_2Epositive \\ & \quad A_{27a}) (ap (ap (c_2Epair_2E_2C (2^{A_{27a}}) (\text{ty_2Epair_2Eprod} (2^{(2^{A_{27a}})})) \\ & \quad (\text{ty_2Erealax_2Ereal}^{(2^{A_{27a}})})) (ap (c_2Emeasure_2Espace A_{27a}) \\ & \quad V0g0)) (ap (ap (c_2Epair_2E_2C (2^{(2^{A_{27a}})})) (\text{ty_2Erealax_2Ereal}^{(2^{A_{27a}})})) \\ & \quad (ap (c_2Emeasure_2Esubsets A_{27a}) V0g0)) V1lam)))))) \Rightarrow (p (ap (c_2Emeasure_2Ealgebra \\ & \quad A_{27a}) (ap (ap (c_2Epair_2E_2C (2^{A_{27a}}) (2^{(2^{A_{27a}})})) (ap (c_2Emeasure_2Espace \\ & \quad A_{27a}) V0g0)) (ap (ap (c_2Emeasure_2Elambda_system A_{27a}) V0g0) \\ & \quad V1lam)))))) \end{aligned}$$