

thm_2Emeasure_2ELAMBDA_SYSTEM_STRONG_ADDITIVE (TMYPjtzCdEtCMvzxr1DCossodpkMmaFY95A)

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Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_21$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ty_2Erealax_2Ereal_REP_CLASS\ V0a)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \tag{5}$$

Definition 6 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal$

Definition 7 We define c_Ebool_EF to be $(ap (c_Ebool_E21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_Emin_E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_EF))$

Definition 10 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Let $c_Emeasure_Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Emeasure_Emeasurable_sets A_27a \in ((2^{(2^{A-27a})})(ty_Epair_Eprod (2^{A-27a}) (ty_Epair_Eprod (2^{(2^{A-27a})}) (ty_Erealax_Ereal^{(2^{A-27a})})))) \quad (6)$$

Let $c_Eenum_EZERO_REP : \iota$ be given. Assume the following.

$$c_Eenum_EZERO_REP \in \omega \quad (7)$$

Let $ty_Eenum_Eenum : \iota$ be given. Assume the following.

$$nonempty ty_Eenum_Eenum \quad (8)$$

Let $c_Eenum_EABS_num : \iota$ be given. Assume the following.

$$c_Eenum_EABS_num \in (ty_Eenum_Eenum^{\omega}) \quad (9)$$

Definition 11 We define c_Eenum_E0 to be $(ap c_Eenum_EABS_num c_Eenum_EZERO_REP)$.

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Eenum_Eenum}) \quad (10)$$

Let $c_Emeasure_Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Emeasure_Emeasure A_27a \in ((2^{(2^{A-27a})})(ty_Epair_Eprod (2^{A-27a}) (ty_Epair_Eprod (2^{(2^{A-27a})}) (ty_Erealax_Ereal^{(2^{A-27a})})))) \quad (11)$$

Definition 12 We define c_Ebool_EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 13 We define $c_Epred_set_EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_EF)$.

Definition 14 We define $c_Ebool_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21 2) (\lambda V2t \in 2.V2t))))$

Definition 15 We define $c_Emeasure_Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_Epair_Eprod (2^{A-27a}) (ty_Erealax_Ereal^{(2^{A-27a})}))$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (12)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (13)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}}) \quad (14)$$

Definition 16 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 17 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))) \quad (15)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (16)$$

Definition 18 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ x\ y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (17)$$

Definition 19 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ s\ t))$

Definition 20 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ s\ t))$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A_27a})})(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))) \quad (18)$$

Definition 21 We define $c_2Emeasure_2Elambda_system$ to be $\lambda A_27a : \iota.\lambda V0gen \in (ty_2Epair_2Eprod\ (2^{A_27a}))$

Definition 22 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ s\ t))$

Definition 23 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1sts \in (2^{(2^{A-27a})})$

Definition 24 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))$

Definition 25 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Emin_2E_40) 2) ((\lambda V2t \in 2.((p V0s) \Rightarrow (p V1t)) \Rightarrow ((p V0s) \Leftrightarrow (p V1t))))$

Definition 26 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))$

Definition 27 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40) 2) ((\lambda V2t \in 2.((p V0P) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0P)) \Rightarrow ((p V0P) \Leftrightarrow (p V1t2))))))$

Definition 28 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Emin_2E_40) 2) ((\lambda V2t \in 2.((p V0s) \Rightarrow (p V1t)) \Rightarrow ((p V0s) \Leftrightarrow (p V1t))))$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{26}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{27}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\neg(\forall V1x \in A_27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p (ap V0P V2x))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a. (p (ap V0P V3x))) \wedge (\forall V4x \in A_27a. (p (ap V1Q V4x))))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((p V0P) \wedge (\forall V2x \in A_27a. (p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p V0P) \wedge (p (ap V1Q V3x))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((\exists V3x \in A_27a. (p (ap V0P V3x))) \vee (\exists V4x \in A_27a. (p (ap V1Q V4x))))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((p V0P) \vee (\exists V2x \in A_27a. (p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A_27a. ((p V0P) \vee (p (ap V1Q V3x))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. ((\exists V2x \in A_27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A_27a. (p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x))))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (41)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1a \in A.27a. ((\exists V2x \in A.27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))) \quad (43)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}). (\forall V4x \in A.27a. (p (ap (ap V0P V4x) (ap V3f V4x)))))) \quad (44)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\
& \quad (2^{A_27a}) (2^{(2^{A_27a})})). ((p (ap (c_2Emeasure_2Ealgebra\ A_27a) \\
V0a)) \Leftrightarrow ((p (ap (ap (c_2Emeasure_2Esubset_class\ A_27a) (ap (c_2Emeasure_2Espace \\
& \quad A_27a) V0a)) (ap (c_2Emeasure_2Esubsets\ A_27a) V0a))) \wedge ((p (ap \\
& \quad (ap (c_2Ebool_2EIN (2^{A_27a})) (c_2Epred_set_2EEMPTY\ A_27a)) \\
& \quad (ap (c_2Emeasure_2Esubsets\ A_27a) V0a))) \wedge ((\forall V1s \in (2^{A_27a}). \\
& \quad ((p (ap (ap (c_2Ebool_2EIN (2^{A_27a})) V1s) (ap (c_2Emeasure_2Esubsets \\
& \quad A_27a) V0a))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN (2^{A_27a})) (ap (ap (c_2Epred_set_2EDIFF \\
& \quad A_27a) (ap (c_2Emeasure_2Espace\ A_27a) V0a)) V1s)) (ap (c_2Emeasure_2Esubsets \\
& \quad A_27a) V0a)))))) \wedge (\forall V2s \in (2^{A_27a}). (\forall V3t \in (2^{A_27a}). \\
& \quad (((p (ap (ap (c_2Ebool_2EIN (2^{A_27a})) V2s) (ap (c_2Emeasure_2Esubsets \\
& \quad A_27a) V0a))) \wedge (p (ap (ap (c_2Ebool_2EIN (2^{A_27a})) V3t) (ap (c_2Emeasure_2Esubsets \\
& \quad A_27a) V0a)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN (2^{A_27a})) (ap (ap (c_2Epred_set_2EINTER \\
& \quad A_27a) V2s) V3t)) (ap (c_2Emeasure_2Esubsets\ A_27a) V0a))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& \quad A_27b. (((ap (ap (c_2Epair_2E_2C\ A_27a\ A_27b) V0x) V1y) = (ap (ap \\
& \quad (c_2Epair_2E_2C\ A_27a\ A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN \\
& \quad A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN\ A_27a) V2x) V1t))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\
& \quad A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) V1v) (ap (c_2Epred_set_2EGSPEC \\
& \quad A_27a\ A_27b) V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap (ap (c_2Epair_2E_2C \\
& \quad A_27a\ 2) V1v) c_2Ebool_2ET) = (ap V0f V2x))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap \\
& \quad (c_2Ebool_2EIN\ A_27a) V0x) (c_2Epred_set_2EEMPTY\ A_27a))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\
& \quad V2x) (ap (ap (c_2Epred_set_2EUNION\ A_27a) V0s) V1t))) \Leftrightarrow ((p (ap \\
& \quad (ap (c_2Ebool_2EIN\ A_27a) V2x) V0s)) \vee (p (ap (ap (c_2Ebool_2EIN \\
& \quad A_27a) V2x) V1t))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c.2Epred_set_2EDIFF\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{62}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a. nonempty A.27a \Rightarrow (\forall V0g0 \in (ty_2Epair_2Eprod \\
& (2^{A.27a}) (2^{(2^{A.27a})})). (\forall V1lam \in (ty_2Erealax_2Ereal^{(2^{A.27a})}). \\
& (\forall V2g \in (2^{A.27a}). (\forall V3l1 \in (2^{A.27a}). (\forall V4l2 \in \\
& (2^{A.27a}). (((p (ap (c_2Emeasure_2Ealgebra A.27a) V0g0)) \wedge ((p \\
& (ap (c_2Emeasure_2Epositive A.27a) (ap (ap (c_2Epair_2E_2C (2^{A.27a}) \\
& (ty_2Epair_2Eprod (2^{(2^{A.27a})}) (ty_2Erealax_2Ereal^{(2^{A.27a})}))) \\
& (ap (c_2Emeasure_2Espace A.27a) V0g0)) (ap (ap (c_2Epair_2E_2C \\
& (2^{(2^{A.27a})}) (ty_2Erealax_2Ereal^{(2^{A.27a})}))) (ap (c_2Emeasure_2Esubsets \\
& A.27a) V0g0)) V1lam)))) \wedge ((p (ap (ap (c_2Ebool_2EIN (2^{A.27a}) \\
& V2g) (ap (c_2Emeasure_2Esubsets A.27a) V0g0))) \wedge ((p (ap (ap (c_2Epred_set_2EDISJOINT \\
& A.27a) V3l1) V4l2)) \wedge ((p (ap (ap (c_2Ebool_2EIN (2^{A.27a}) V3l1) \\
& (ap (ap (c_2Emeasure_2Elambda_system A.27a) V0g0) V1lam)))) \wedge \\
& (p (ap (ap (c_2Ebool_2EIN (2^{A.27a}) V4l2) (ap (ap (c_2Emeasure_2Elambda_system \\
& A.27a) V0g0) V1lam))))))))) \Rightarrow ((ap V1lam (ap (ap (c_2Epred_set_2EINTER \\
& A.27a) (ap (ap (c_2Epred_set_2EUNION A.27a) V3l1) V4l2)) V2g)) = \\
& (ap (ap (c_2Erealax_2Ereal_add (ap V1lam (ap (ap (c_2Epred_set_2EINTER \\
& A.27a) V3l1) V2g))) (ap V1lam (ap (ap (c_2Epred_set_2EINTER A.27a) \\
& V4l2) V2g)))))))))
\end{aligned}$$