

thm_2Emeasure_2EMEASURABLE__COMP (TMazSte3W3RToPXvUiCJ1LVAKstqy9pnnsL)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2EBOUNDED` to be $(\lambda V0v \in 2. \text{c_2Ebool_2ET})$.

Definition 4 We define `c_2Ebool_2EIN` to be $\lambda A. \lambda a : \iota. (\lambda V0x \in A. \lambda a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f V0x)))$

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Ebool_2E_3F` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Epair_2EABS_prod } A_27a A_27b \in ((\text{ty_2Epair_2Eprod } A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

Definition 10 We define `c_2Epair_2E_2C` to be $\lambda A. \lambda a : \iota. \lambda A_27b : \iota. \lambda V0x \in A. \lambda V1y \in A_27b. (\text{ap } (\text{c_2Emin_2E_40 } A))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (3)$$

Definition 11 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ V0P))$

Definition 12 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EBIGUNION\ A_27a\ V0s\ V1t))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (4)$$

Definition 13 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 14 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b})$

Definition 15 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_3F\ A_27a\ V0s))$

Definition 16 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 18 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2ESUBSET\ A_27a\ V0s\ V1t))$

Definition 19 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 20 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (5)$$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (6)$$

Definition 21 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 22 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EBIGUNION\ A_27a\ V0s\ V1t))$

Definition 23 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 24 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 25 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b}).(ap\ (c_2Epred_set_2EBIGUNION\ A_27a\ V0f\ V1s))$

Definition 26 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EINTER) V0s V1t)$

Definition 27 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).(ap (c_2Epred_set_2EFUNSET) V0P V1Q)$

Definition 28 We define $c_2Emeasure_2E measurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Epr. (c_2Emeasure_2E measurable) A_27a A_27b V0a)$

Definition 29 We define $c_2E combin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27a}).(ap (c_2E combin_2Eo) V0f V1g)$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{9}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \tag{13}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27}))))))
\end{aligned} \tag{18}$$

Assume the following.

$$(\forall V0v \in 2.((p (ap c_2Ebool_2EBOUNDED V0v)) \Leftrightarrow True)) \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\
& nonempty A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\
& (\forall V2x \in A_27c.((ap (ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) \\
& V0f) V1g) V2x) = (ap V0f (ap V1g V2x))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (2^{(2^{A_27a})})).((p (ap (c_2Emeasure_2Esigma_algebra \\
& A_27a) V0p)) \Leftrightarrow ((p (ap (ap (c_2Emeasure_2Esubset_class A_27a) \\
& (ap (c_2Emeasure_2Espace A_27a) V0p)) (ap (c_2Emeasure_2Esubsets \\
& A_27a) V0p))) \wedge ((p (ap (ap (c_2Ebool_2EIN (2^{A_27a}) (c_2Epred_set_2EEMPTY \\
& A_27a) (ap (c_2Emeasure_2Esubsets A_27a) V0p))) \wedge ((\forall V1s \in \\
& (2^{A_27a}).((p (ap (ap (c_2Ebool_2EIN (2^{A_27a}) V1s) (ap (c_2Emeasure_2Esubsets \\
& A_27a) V0p))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN (2^{A_27a}) (ap (ap (c_2Epred_set_2EDIFF \\
& A_27a) (ap (c_2Emeasure_2Espace A_27a) V0p)) V1s) (ap (c_2Emeasure_2Esubsets \\
& A_27a) V0p)))))) \wedge (\forall V2c \in (2^{(2^{A_27a})}).(((p (ap (c_2Epred_set_2Ecountable \\
& (2^{A_27a}) V2c) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET (2^{A_27a}) \\
& V2c) (ap (c_2Emeasure_2Esubsets A_27a) V0p)))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN \\
& (2^{A_27a}) (ap (c_2Epred_set_2EBIGUNION A_27a) V2c) (ap (c_2Emeasure_2Esubsets \\
& A_27a) V0p)))))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0a \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})})). (\forall V1b \in \\
& \quad (ty_2Epair_2Eprod\ (2^{A.27b})\ (2^{(2^{A.27b})})). (\forall V2f \in (A.27b^{A.27a}). \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A.27b^{A.27a})\ V2f)\ (ap\ (ap\ (c_2Emeasure_2Emeasurable \\
& \quad A.27a\ A.27b)\ V0a)\ V1b)))) \Leftrightarrow ((p\ (ap\ (c_2Emeasure_2Esigma_algebra \\
& \quad A.27a)\ V0a)) \wedge ((p\ (ap\ (c_2Emeasure_2Esigma_algebra\ A.27b)\ V1b)) \wedge \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A.27b^{A.27a})\ V2f)\ (ap\ (ap\ (c_2Epred_set_2EFUNSET \\
& \quad A.27a\ A.27b)\ (ap\ (c_2Emeasure_2Espace\ A.27a)\ V0a))\ (ap\ (c_2Emeasure_2Espace \\
& \quad A.27b)\ V1b)))) \wedge (\forall V3s \in (2^{A.27b}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (2^{A.27b})\ V3s)\ (ap\ (c_2Emeasure_2Esubsets\ A.27b)\ V1b)))) \Rightarrow (p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a})\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad A.27a)\ (ap\ (ap\ (c_2Epred_set_2EPREIMAGE\ A.27a\ A.27b)\ V2f)\ V3s)) \\
& \quad (ap\ (c_2Emeasure_2Espace\ A.27a)\ V0a)))\ (ap\ (c_2Emeasure_2Esubsets \\
& \quad A.27a)\ V0a))))))))) \\
& \hspace{15em} (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ V1t)))))) \\
& \hspace{15em} (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a) \\
& \quad V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (24) \\
& \quad (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27a)\ V2x)\ V1t))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27a}). (\forall V2Q \in \\
& \quad (2^{A.27b}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A.27b^{A.27a})\ V0f)\ (ap\ (ap \\
& \quad (c_2Epred_set_2EFUNSET\ A.27a\ A.27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\
& \quad A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27b)\ (ap\ V0f\ V3x))\ V2Q)))))) \\
& \hspace{15em} (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (A.27b^{A.27a}). (\forall V1s \in (2^{A.27b}). (\forall V2x \in \\
& \quad A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EPREIMAGE \\
& \quad A.27a\ A.27b)\ V0f)\ V1s))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27b)\ (ap\ V0f \\
& \quad V2x))\ V1s)))))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}). (\forall V1g \in (A.27c^{A.27b}). \\
& (\forall V2s \in (2^{A.27c}). ((ap\ (ap\ (c.2Epred_set_2EPREIMAGE\ A.27a \\
& A.27b)\ V0f)\ (ap\ (ap\ (c.2Epred_set_2EPREIMAGE\ A.27b\ A.27c)\ V1g) \\
& V2s)) = (ap\ (ap\ (c.2Epred_set_2EPREIMAGE\ A.27a\ A.27c)\ (ap\ (ap\ (\\
& c.2Ecombin_2Eo\ A.27a\ A.27c\ A.27b)\ V1g)\ V0f))\ V2s))))))
\end{aligned} \tag{27}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{31}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False))) \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\
& ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\\
& \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (41)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c. \\ & nonempty A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27c^{A.27b}). \\ & (\forall V2a \in (ty_2Epair_2Eprod (2^{A.27a}) (2^{(2^{A.27a})})).(\forall V3b \in \\ & (ty_2Epair_2Eprod (2^{A.27b}) (2^{(2^{A.27b})})).(\forall V4c \in (ty_2Epair_2Eprod \\ & (2^{A.27c}) (2^{(2^{A.27c})})).(((p (ap (ap (c.2Ebool_2EIN (A.27b^{A.27a}) \\ & V0f) (ap (ap (c.2Emeasure_2Emeasurable A.27a A.27b) V2a) V3b)))) \wedge \\ & (p (ap (ap (c.2Ebool_2EIN (A.27c^{A.27b}) V1g) (ap (ap (c.2Emeasure_2Emeasurable \\ & A.27b A.27c) V3b) V4c)))))) \Rightarrow (p (ap (ap (c.2Ebool_2EIN (A.27c^{A.27a}) \\ & (ap (ap (c.2Ecombin_2Eo A.27a A.27c A.27b) V1g) V0f) (ap (ap (c.2Emeasure_2Emeasurable \\ & A.27a A.27c) V2a) V4c)))))))))) \end{aligned}$$