# thm_2Emeasure_2EMEASURABLE__COMP__STRONGER (TMXQuRKg8nPshnNSo92eac94j3B2D7q4M9N)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.\mathsf{c\_2Ebool\_2ET})$.

**Definition 4** *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 5** *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

**Definition 6** *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A\_$

**Definition 7** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 8** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 9** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 10** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

**Definition 11** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 12** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

**Definition 13** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (3)$$

**Definition 14** *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (4)$$

**Definition 15** *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.$c_2Ebool_2ET$)$.

**Definition 16** *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

**Definition 17** *We define* c_2Epred__set_2Ecountable *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_3F

**Definition 18** *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

**Definition 19** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

**Definition 20** *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

**Definition 21** *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.$c_2Ebool_2EF$)$.

**Definition 22** *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

**Definition 23** *We define* c_2Emeasure_2Esubset__class *to be* $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27}}$

Let $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (5)$$

Let $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (6)$$

**Definition 24** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

**Definition 25** *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

**Definition 26** *We define* c_2Emeasure_2Ealgebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2}$

**Definition 27** *We define* c_2Emeasure_2Esigma__algebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_2}$

**Definition 28** *We define* c_2Epred__set_2EPREIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V$

**Definition 29** *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

**Definition 30** *We define* c_2Epred__set_2EFUNSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in ($

**Definition 31** *We define* c_2Emeasure_2Emeasurable *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (ty\_2Epair\_2Epro$

**Definition 32** *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

**Definition 33** *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \land (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \land (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{13}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \land (((\neg True) \Leftrightarrow False) \land ((\neg False) \Leftrightarrow True))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \vee (p\ V1Q)) \Leftrightarrow (\exists V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \vee (\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{21}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{22}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{23}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \vee (\neg (p\ V1B)))) \wedge ((\neg ((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \wedge (\neg (p\ V1B))))))) \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{25}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow( \forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y))))\Leftrightarrow(\exists V3f \in (A\_27b^{A\_27a}).( \forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \tag{27}$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v))\Leftrightarrow True)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}). (\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow\forall A\_27d.nonempty\ A\_27d\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27a^{A\_27c}).(\forall V2h \in (A\_27c^{A\_27d}).((ap\ ( ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27d\ A\_27b\ A\_27a)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27d\ A\_27a\ A\_27c)\ V1g)\ V2h)) = (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27d\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g))\ V2h))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V0p)) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Esubset\_\_class}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0p))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p))) \wedge ((\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V1s)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0p))\ V1s))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p))))) \wedge (\forall V2c \in (2^{(2^{A\_27a})}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2Ecountable}\ (2^{A\_27a}))\ V2c)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (2^{A\_27a}))\ V2c)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ A\_27a)\ V2c))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p)))))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).(\forall V1b \in (ty\_2Epair\_2Eprod\ (2^{A\_27b})\ (2^{(2^{A\_27b})})).(\forall V2f \in (A\_27b^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V2f)\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ A\_27b)\ V0a)\ V1b))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V0a)) \wedge ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27b)\ V1b)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V2f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ A\_27b)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0a))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27b)\ V1b)))) \wedge (\forall V3s \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27b}))\ V3s)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27b)\ V1b))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPREIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V3s))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0a)))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a)))))))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V0y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s)))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0f \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27a}).(\forall V2Q \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ A\_27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0f\ V3x))\ V2Q)))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0f \in (A\_27b^{A\_27a}).(\forall V1s \in (2^{A\_27b}).((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPREIMAGE}\ A\_27a\ A\_27b)\ V0f)\ V1s) = (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ 2\ A\_27b)\ V1s)\ V0f)))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0f \in (A\_27b^{A\_27a}).(\forall V1s \in (2^{A\_27b}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPREIMAGE}\ A\_27a\ A\_27b)\ V0f)\ V1s))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0f\ V2x))\ V1s)))))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{41}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{43}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{44}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{49}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27c^{A\_27b}).\ (\forall V2a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).(\forall V3b \in (ty\_2Epair\_2Eprod\ (2^{A\_27b})\ (2^{(2^{A\_27b})})).(\forall V4c \in (ty\_2Epair\_2Eprod\ (2^{A\_27c})\ (2^{(2^{A\_27c})})).(\forall V5t \in (2^{A\_27b}).(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ A\_27b)\ V2a)\ V3b))) \wedge ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27c)\ V4c)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27c^{A\_27b}))\ V1g)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27b\ A\_27c)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27b)\ V3b))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27c)\ V4c)))) \wedge ((p\ (ap\ (\ ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V0f)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V2a)))\ V5t)) \wedge (\forall V6s \in (2^{A\_27c}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27c}))\ V6s)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27c)\ V4c))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27b}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPREIMAGE}\ A\_27b\ A\_27c)\ V1g)\ V6s))\ V5t))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27b)\ V3b))))))))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27c^{A\_27a}))\ (ap\ (ap\ (\ \mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27c\ A\_27b)\ V1g)\ V0f))\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ A\_27c)\ V2a)\ V4c))))))))))$$