

# thm\_2Emeasure\_2EMEASURABLE\_\_RANGE\_\_REDUCE (TMZ1RWrw6X587ME9SHAYDkNw5oPYSvDXMrv)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets\ A\_27a \in ((2^{(2^{A\_27a})})(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \tag{3}$$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in ((2^{A\_27a})(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \tag{4}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{5}$$

**Definition 6** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c\_2Ebool\_2ET)$ .

**Definition 7** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A.27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap\ V1f\ V0x)))$

**Definition 8** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A.27a\ A.27b \in ((ty\_2Epair\_2Eprod\ A.27a\ A.27b)^{(2^{A-27b})^{A-27a}}) \end{aligned} \quad (6)$$

**Definition 10** We define  $c\_2Epair\_2E2C$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap\ (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A.27a\ A.27b \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod\ A.27a\ 2)^{A-27b}}) \end{aligned} \quad (7)$$

**Definition 11** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0f \in (A.27b^{A-27a}).\lambda V1s \in$

**Definition 12** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E40$

**Definition 14** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A.27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap\ (c\_2Epred\_set\_2E$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow c\_2Emeasure\_2Emeasure\ A.27a \in ( \\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})})(ty\_2Epair\_2Eprod\ (2^{A-27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))) \end{aligned} \quad (8)$$

**Definition 15** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in (A.27b^{A-27c}).\lambda V1$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega^{omega}}) \quad (10)$$

**Definition 16** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum})})^{(ty\_2Epair\_2Eprod\ ty\_2Eenum\_2Eenum)}) \quad (11)$$

**Definition 17** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E7E))$

Let  $c\_2Eenum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EREP\_num \in (\omega^{ty\_2Eenum\_2Eenum}) \quad (12)$$

Let  $c\_2Eenum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Eenum\_2ESUC\_REP \in (\omega^{\omega}) \quad (13)$$

**Definition 18** We define  $c\_2Eenum\_2ESUC$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.(ap\ c\_2Eenum\_2EABS\_num)$

**Definition 19** We define  $c\_2Eprim\_rec\_2E3C$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

**Definition 20** We define  $c\_2Earithmetic\_2E3E$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

**Definition 21** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2)))$

**Definition 22** We define  $c\_2Earithmetic\_2E3E\_3D$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (14)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (15)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40\ t))$

Let  $c\_2Erealax\_2Etreal\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (16)$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (17)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (18)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 25** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (19)$$

**Definition 26** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 27** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (20)$$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (21)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 29** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 30** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 31** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (22)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (23)$$

**Definition 32** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (24)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (25)$$



Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})})) \quad (31)$$

**Definition 46** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2E$

**Definition 47** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 48** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 49** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 50** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 51** We define  $c\_2Epred\_set\_2EPREIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V$

**Definition 52** We define  $c\_2Emeasure\_2Emeasurable$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 53** We define  $c\_2Epred\_set\_2EPOW$  to be  $\lambda A\_27a : \iota.\lambda V0set \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2E$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge (((\neg False) \Leftrightarrow True)))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in \\ & (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ & (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in \\ & (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ & (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}).(p\ (ap \\ & (c\_2Emeasure\_2Esigma\_algebra\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ & (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0sp)\ (ap\ (c\_2Epred\_set\_2EPOW\ A\_27a) \\ & V0sp)))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad \forall V0a \in (ty\_2Epair\_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})})). (\forall V1b \in \\
& \quad (ty\_2Epair\_2Eprod\ (2^{A.27b})\ (2^{(2^{A.27b})})). (\forall V2f \in (A.27b^{A.27a}). \\
& \quad ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A.27b^{A.27a})\ V2f)\ (ap\ (ap\ (c\_2Emeasure\_2E measurable \\
& \quad A.27a\ A.27b)\ V0a)\ V1b))) \Leftrightarrow ((p\ (ap\ (c\_2Emeasure\_2E sigma\_algebra \\
& \quad A.27a)\ V0a)) \wedge ((p\ (ap\ (c\_2Emeasure\_2E sigma\_algebra\ A.27b)\ V1b)) \wedge \\
& \quad ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A.27b^{A.27a})\ V2f)\ (ap\ (ap\ (c\_2Epred\_set\_2EFUNSET \\
& \quad A.27a\ A.27b)\ (ap\ (c\_2Emeasure\_2Espace\ A.27a)\ V0a)\ (ap\ (c\_2Emeasure\_2Espace \\
& \quad A.27b)\ V1b)))))) \wedge (\forall V3s \in (2^{A.27b}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad (2^{A.27b})\ V3s)\ (ap\ (c\_2Emeasure\_2Esubsets\ A.27b)\ V1b))) \Rightarrow (p\ ( \\
& \quad ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A.27a})\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER \\
& \quad A.27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EPREIMAGE\ A.27a\ A.27b)\ V2f)\ V3s)) \\
& \quad (ap\ (c\_2Emeasure\_2Espace\ A.27a)\ V0a)))\ (ap\ (c\_2Emeasure\_2Esubsets \\
& \quad A.27a)\ V0a)))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ \\
& \quad V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ ( \\
& \quad (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). (\forall V2u \in (2^{A.27a}). ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET \\
& \quad A.27a)\ V0s)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A.27a)\ V1t)\ V2u))) \Leftrightarrow \\
& \quad ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A.27a)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ \\
& \quad (c\_2Epred\_set\_2ESUBSET\ A.27a)\ V0s)\ V2u))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad \forall V0y \in A.27b. (\forall V1s \in (2^{A.27a}). (\forall V2f \in (A.27b^{A.27a}). \\
& \quad ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& \quad A.27a\ A.27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A.27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\
& \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V3x)\ V1s))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad \forall V0f \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27a}). (\forall V2Q \in \\
& \quad (2^{A.27b}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A.27b^{A.27a})\ V0f)\ (ap\ (ap \\
& \quad (c\_2Epred\_set\_2EFUNSET\ A.27a\ A.27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\
& \quad A.27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A.27b)\ (ap\ V0f\ V3x)\ V2Q))))))
\end{aligned} \tag{50}$$



Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0set \in (2^{A-27a}). (\forall V1e \in \\ & (2^{A-27a}). ((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ (2^{A-27a})\ V1e)\ (ap\ (c.2Epred\_set\_2EPOW \\ & A.27a)\ V0set))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Epred\_set\_2ESUBSET\ A.27a)\ V1e) \\ & V0set)))))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (55)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg( \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (62)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\ & \forall V0m \in (ty\_2Epair\_2Eprod \ (2^{A\_27a}) \ (ty\_2Epair\_2Eprod \ ( \\ & \quad 2^{(2^{A\_27a})}) \ (ty\_2Erealax\_2Ereal(2^{A\_27a}))))). (\forall V1f \in \\ & (A\_27b^{A\_27a}). (\forall V2s \in (2^{A\_27b}). (((p \ (ap \ (c\_2Emeasure\_2Emeasure\_space \\ & \quad A\_27a) \ V0m)) \wedge ((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ (A\_27b^{A\_27a}) \ V1f) \ (ap \\ & \quad (ap \ (c\_2Emeasure\_2Emeasurable \ A\_27a \ A\_27b) \ (ap \ (ap \ (c\_2Epair\_2E\_2C \\ & \quad (2^{A\_27a}) \ (2^{(2^{A\_27a})})) \ (ap \ (c\_2Emeasure\_2Em\_space \ A\_27a) \\ & \quad V0m)) \ (ap \ (c\_2Emeasure\_2Emeasurable\_sets \ A\_27a) \ V0m)))) \ (ap \ ( \\ & \quad ap \ (c\_2Epair\_2E\_2C \ (2^{A\_27b}) \ (2^{(2^{A\_27b})})) \ V2s) \ (ap \ (c\_2Epred\_set\_2EPOW \\ & \quad A\_27b) \ V2s)))))) \wedge (\neg(V2s = (c\_2Epred\_set\_2EEMPTY \ A\_27b)))))) \Rightarrow \\ & (p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ (A\_27b^{A\_27a}) \ V1f) \ (ap \ (ap \ (c\_2Emeasure\_2Emeasurable \\ & \quad A\_27a \ A\_27b) \ (ap \ (ap \ (c\_2Epair\_2E\_2C \ (2^{A\_27a}) \ (2^{(2^{A\_27a})})) \\ & \quad (ap \ (c\_2Emeasure\_2Em\_space \ A\_27a) \ V0m)) \ (ap \ (c\_2Emeasure\_2Emeasurable\_sets \\ & \quad A\_27a) \ V0m)))) \ (ap \ (ap \ (c\_2Epair\_2E\_2C \ (2^{A\_27b}) \ (2^{(2^{A\_27b})})) \\ & \quad (ap \ (ap \ (c\_2Epred\_set\_2EINTER \ A\_27b) \ V2s) \ (ap \ (ap \ (c\_2Epred\_set\_2EIMAGE \\ & \quad A\_27a \ A\_27b) \ V1f) \ (ap \ (c\_2Emeasure\_2Em\_space \ A\_27a) \ V0m)))))) \ ( \\ & \quad ap \ (c\_2Epred\_set\_2EPOW \ A\_27b) \ (ap \ (ap \ (c\_2Epred\_set\_2EINTER \\ & \quad A\_27b) \ V2s) \ (ap \ (ap \ (c\_2Epred\_set\_2EIMAGE \ A\_27a \ A\_27b) \ V1f) \ (ap \\ & \quad (c\_2Emeasure\_2Em\_space \ A\_27a) \ V0m))))))))) \end{aligned}$$