

thm_2Emeasure_2EMEASURABLE_SIGMA_PREIMAGES (TMEifoC9ChnZcgCbrRJBqHSedLerUU8kkbL)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 4 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (V1t2))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod A_27a A_27b) (V0x V1y))$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 11 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap$

Definition 12 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A_27a}). \lambda V1sts \in (2^{(2^{A_27a})})$

Definition 13 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2^{A_27b})}) \quad (3)$$

Definition 15 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap (c_2Epred_set_2EGSPEC$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (4)$$

Definition 16 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_2ET)$.

Definition 17 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27b})$

Definition 18 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E_3F$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Emeasure_2Esubsets A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})})) \quad (5)$$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Emeasure_2Espace A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})})) \quad (6)$$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (c_2Ebool_2E_5C_2F V0t1 V1t2 V2t))))$

Definition 20 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2E_5C_2F$

Definition 21 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2. V0t))$.

Definition 22 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap (c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_5C_2F$

Definition 23 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2E_7E$

Definition 24 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 25 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 26 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}))$

Definition 27 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b})$

Definition 28 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b})$

Definition 29 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b})$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee \neg(p V0t))) \tag{10}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (30)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0P \in ((2^{A_27b})^{A_27a}). ((\forall V1x \in A_27a. (\exists V2y \in A_27b. (p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_27b)^{A_27a}). (\forall V4x \in A_27a. (p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}). (\forall V1y \in (2^{(2^{A_27a})}). ((ap\ (c_2Emeasure_2Espace\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (2^{(2^{A_27a})}))\ V0x)\ V1y)) = V0x))) \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (2^{A.27a}).(\forall V1y \in \\ & (2^{(2^{A.27a})}).((ap\ (c.2Emeasure.2Esubsets\ A.27a)\ (ap\ (ap\ (c.2Epair.2E.2C \\ & (2^{A.27a})\ (2^{(2^{A.27a})}))\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty.2Epair.2Eprod \\ & (2^{A.27a})\ (2^{(2^{A.27a})})).((p\ (ap\ (c.2Emeasure.2Esigma_algebra \\ & A.27a)\ V0p)) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Emeasure.2Esub_class\ A.27a) \\ & (ap\ (c.2Emeasure.2Espace\ A.27a)\ V0p))\ (ap\ (c.2Emeasure.2Esubsets \\ & A.27a)\ V0p))) \wedge ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ (c.2Epred_set.2EEMPTY \\ & A.27a))\ (ap\ (c.2Emeasure.2Esubsets\ A.27a)\ V0p))) \wedge ((\forall V1s \in \\ & (2^{A.27a}).((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ V1s)\ (ap\ (c.2Emeasure.2Esubsets \\ & A.27a)\ V0p))) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ (ap\ (ap\ (c.2Epred_set.2EDIFF \\ & A.27a)\ (ap\ (c.2Emeasure.2Espace\ A.27a)\ V0p))\ V1s))\ (ap\ (c.2Emeasure.2Esubsets \\ & A.27a)\ V0p)))))) \wedge (\forall V2c \in (2^{(2^{A.27a})}).(((p\ (ap\ (c.2Epred_set.2Ecountable \\ & (2^{A.27a})\ V2c)) \wedge (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ (2^{A.27a}) \\ & V2c)\ (ap\ (c.2Emeasure.2Esubsets\ A.27a)\ V0p)))))) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & (2^{A.27a})\ (ap\ (c.2Epred_set.2EBIGUNION\ A.27a)\ V2c))\ (ap\ (c.2Emeasure.2Esubsets \\ & A.27a)\ V0p)))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0a \in (ty.2Epair.2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})})).(\forall V1b \in \\ & (ty.2Epair.2Eprod\ (2^{A.27b})\ (2^{(2^{A.27b})})).(\forall V2f \in (A.27b^{A.27a}). \\ & (((p\ (ap\ (c.2Emeasure.2Esigma_algebra\ A.27a)\ V0a)) \wedge ((p\ (ap\ (\\ & c.2Emeasure.2Esigma_algebra\ A.27b)\ V1b)) \wedge ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & (A.27b^{A.27a})\ V2f)\ (ap\ (ap\ (c.2Epred_set.2EFUNSET\ A.27a\ A.27b) \\ & (ap\ (c.2Emeasure.2Espace\ A.27a)\ V0a))\ (ap\ (c.2Emeasure.2Espace \\ & A.27b)\ V1b)))))) \wedge (\forall V3s \in (2^{A.27b}).((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & (2^{A.27b})\ V3s)\ (ap\ (c.2Emeasure.2Esubsets\ A.27b)\ V1b))) \Rightarrow (p\ (\\ & ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ (ap\ (ap\ (c.2Epred_set.2EPREIMAGE \\ & A.27a\ A.27b)\ V2f)\ V3s))\ (ap\ (c.2Emeasure.2Esubsets\ A.27a)\ V0a)))))) \Rightarrow \\ & (\forall V4c \in (2^{(2^{A.27a})}).(((p\ (ap\ (c.2Epred_set.2Ecountable \\ & (2^{A.27a})\ V4c)) \wedge (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ (2^{A.27a}) \\ & V4c)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ (2^{A.27b})\ (2^{A.27a}))\ (ap\ (\\ & c.2Epred_set.2EPREIMAGE\ A.27a\ A.27b)\ V2f))\ (ap\ (c.2Emeasure.2Esubsets \\ & A.27b)\ V1b)))))) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ (ap\ (c.2Epred_set.2EBIGUNION \\ & A.27a\ V4c))\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ (2^{A.27b})\ (2^{A.27a})) \\ & (ap\ (c.2Epred_set.2EPREIMAGE\ A.27a\ A.27b)\ V2f))\ (ap\ (c.2Emeasure.2Esubsets \\ & A.27b)\ V1b)))))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V0s)\ V1t)) \wedge \\ & (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V1t)\ V0s))) \Rightarrow (V0s = V1t)))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (\\ & ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (\neg (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27a}). (\forall V2Q \in \\ & (2^{A_27b}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (A_27b^{A_27a})\ V0f)\ (ap\ (ap \\ & (c_2Epred_set_2EFUNSET\ A_27a\ A_27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\ & A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ (ap\ V0f\ V3x))\ V2Q)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27b}). (\forall V2x \in \\ & A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EPREIMAGE \\ & A_27a\ A_27b)\ V0f)\ V1s))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ V0f \\ & V2x))\ V1s)))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EPREIMAGE \\ & A_27a\ A_27b)\ V0f)\ (c_2Epred_set_2EEMPTY\ A_27b)) = (c_2Epred_set_2EEMPTY \\ & A_27a)) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (A.27b^{A.27a}). (\forall V1s \in (2^{A.27b}). (\forall V2t \in \\
& \quad (2^{A.27b}). ((ap\ (ap\ (c.2Epred_set_2EPREIMAGE\ A.27a\ A.27b)\ V0f) \\
& \quad (ap\ (ap\ (c.2Epred_set_2EDIFF\ A.27b)\ V1s)\ V2t)) = (ap\ (ap\ (c.2Epred_set_2EDIFF \\
& \quad A.27a)\ (ap\ (ap\ (c.2Epred_set_2EPREIMAGE\ A.27a\ A.27b)\ V0f)\ V1s)) \\
& \quad (ap\ (ap\ (c.2Epred_set_2EPREIMAGE\ A.27a\ A.27b)\ V0f)\ V2t)))))) \\
& \hspace{15em} (44)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (46)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (48)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (49)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee (\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \\
& \hspace{15em} (50)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q)) \vee (\neg(p\ V2r))) \wedge (((p\ V1q) \vee \\
& \quad (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \\
& \hspace{15em} (51)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\
& \quad ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \\
& \hspace{15em} (52)
\end{aligned}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (59)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \forall V0a \in (ty_2Epair_2Eprod \ (2^{A_27a}) \ (2^{(2^{A_27a})})). (\forall V1b \in \\ & (ty_2Epair_2Eprod \ (2^{A_27b}) \ (2^{(2^{A_27b})})). (\forall V2f \in (A_27b^{A_27a}). \\ & (((p \ (ap \ (c_2Emeasure_2Esigma_algebra \ A_27a) \ V0a)) \wedge ((p \ (ap \ (\\ & c_2Emeasure_2Esigma_algebra \ A_27b) \ V1b)) \wedge ((p \ (ap \ (c_2Ebool_2EIN \\ & (A_27b^{A_27a}) \ V2f) \ (ap \ (ap \ (c_2Epred_set_2EFUNSET \ A_27a \ A_27b) \\ & (ap \ (c_2Emeasure_2Espace \ A_27a) \ V0a)) \ (ap \ (c_2Emeasure_2Espace \\ & A_27b) \ V1b)))) \wedge (\forall V3s \in (2^{A_27b}). ((p \ (ap \ (ap \ (c_2Ebool_2EIN \\ & (2^{A_27b}) \ V3s) \ (ap \ (c_2Emeasure_2Esubsets \ A_27b) \ V1b))) \Rightarrow (p \ (\\ & ap \ (ap \ (c_2Ebool_2EIN \ (2^{A_27a}) \ (ap \ (ap \ (c_2Epred_set_2EPREIMAGE \\ & A_27a \ A_27b) \ V2f) \ V3s)) \ (ap \ (c_2Emeasure_2Esubsets \ A_27a) \ V0a)))))) \Rightarrow \\ & (p \ (ap \ (c_2Emeasure_2Esigma_algebra \ A_27a) \ (ap \ (ap \ (c_2Epair_2E2C \\ & (2^{A_27a}) \ (2^{(2^{A_27a})}))) \ (ap \ (c_2Emeasure_2Espace \ A_27a) \ V0a)) \\ & (ap \ (ap \ (c_2Epred_set_2EIMAGE \ (2^{A_27b}) \ (2^{A_27a})) \ (ap \ (c_2Epred_set_2EPREIMAGE \\ & A_27a \ A_27b) \ V2f)) \ (ap \ (c_2Emeasure_2Esubsets \ A_27b) \ V1b)))))) \end{aligned}$$